PROCEEDINGS OF SCIENCE

ROUND TABLE DISCUSSION
Correlations and Fluctuations in Nuclear Collisions

Scientific Secretary: Peter Steinberg
Brookhaven National Laborarory
Upton, NY 11973, USA
E-mail: peter.steinberg@bnl.gov

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1. Item 1: Wojtek Broniowski (WB)

One difficulty in comparing results of event-by-event momentum fluctuations presented by various experimental groups is the multitude of measures used. Here we briefly show how under assumptions of 1) small dynamical compared to statistical fluctuations and 2) sharp distribution in the multiplicity variable these measures are simply proportional to the *covariance*. Although these remarks are perhaps obvious to practitioners in the field, they seem worth reminding, as discussions showed confusion.

Suppose we have events of class n (formally, this can be any number chracteristic of the event: the multiplicity of detected particles, the number of participants, the response of a given detector, *etc.*) distributed according to the probability distribution P_n. Let $\rho_n(p_1,\ldots,p_n)$ denote the n-particle distribution of variables p_i *within events of class n, e.g*, the distribution of transverse momenta in events of a fixed multiplicity n. The subscript $_n$ indicates that ρ depends functionally on n. The full probability distribution of obtaining event of class n with momenta $p_1,\ldots,p_n$ is

$$P_n\rho_n(p_1,\ldots,p_n). \tag{1.1}$$

The *marginal* probability distributions are obtained from $\rho_n(p_1,\ldots,p_n)$ by integrating over k momenta,

$$\rho_n(p_1,\ldots,p_{n-k}) = \int dp_{n-k+1}\ldots dp_n\rho_n(p_1,\ldots,p_n).$$

Next, we introduce the relevant moments for the distributions of class n:

$$\begin{aligned}
\overline{p_n} &= \int dp\rho_n(p)p, \\
\sigma_n^2(p) &= \int dp\rho_n(p)(p-\overline{p_n})^2, \\
\mathrm{cov}_n(p_1,p_2) &= \int dp_1dp_2\rho_n(p_1,p_2)(p_1-\overline{p_n})(p_2-\overline{p_n}),
\end{aligned} \tag{1.2}$$

where $\rho_n(p)$ and $\rho_n(p_1,p_2)$ are the one- and two-particle *marginal* distributions within the class n. Now, in a typical setup we are interested in broader classes, containing n in the range $n_1 \le n \le n_2$. We denote for brevity $\sum_n = \sum_{n=n_1}^{n_2}$.

Let us illustrate the basic statistical facts on the example of the measures σ^2_{dyn} and F_{p_T}. For other measures the analysis is analogous. Consider the variable $M_n = (p_1+\ldots+p_n)/n$, *i.e.* the average value of the variable p. Then

$$\begin{aligned}
\overline{M} &= \sum_n P_n \int dp_1\ldots dp_n\rho_n(p_1,\ldots,p_n)\frac{p_1+\ldots+p_n}{n} = \sum_n P_n\overline{p_n}, \\
\overline{M^2} &= \sum_n P_n \int dp_1\ldots dp_n\rho_n(p_1,\ldots,p_n)\frac{1}{n^2}\sum_{i,j=1}^{n}\left[(p_i-\overline{p_n})(p_j-\overline{p_n})+\overline{p_n}^2\right], \\
\sigma_M^2 &= \overline{M^2}-\overline{M}^2 = \sum_n P_n\overline{p_n}^2 - \left(\sum_n P_n\overline{p_n}\right)^2 + \sum_n P_n\frac{\sigma_n^2(p)}{n} + \sum_n P_n\frac{1}{n^2}\sum_{i\neq j=1}^{n}\mathrm{cov}_n(p_i,p_j).
\end{aligned} \tag{1.3}$$

Suppose mixing of events is performed. Then, by definition, no correlations are present, $\mathrm{cov}_n^{\mathrm{mix}}(p_i,p_j)=0$, and

$$\sigma_M^{2,\mathrm{mix}} = \sum_n P_n\overline{p_n}^2 - \left(\sum_n P_n\overline{p_n}\right)^2 + \sum_n P_n\frac{\sigma_n^2(p)}{n}. \tag{1.4}$$

By definition, $\sigma^2_{\text{dyn}} = \sigma^2_M - \sigma^{2,\text{mix}}_M$. With above results

$$\sigma^2_{\text{dyn}} = \sum_n P_n \frac{1}{n^2} \sum_{i \neq j} \text{cov}_n(p_i, p_j). \tag{1.5}$$

Note that all above results are *exact*, just following from obvious manipulations. Thus σ^2_{dyn} is a *weighted sum of total (summed over particle pairs) covariances at fixed n*, $\sum_{i\neq j} \text{cov}_n(p_i, p_j)$, with weights equal to $P_n \frac{1}{n^2}$. Moreover, all quantities in Eq.(1.3) or (1.4) are possible to obtain experimentally from the given event sample.

Now let us have a look on $F_{p_T} \equiv (\sqrt{\omega} - \sqrt{\omega_{\text{mix}}})/\sqrt{\omega_{\text{mix}}}$, where $\omega = \sigma^2_M/\overline{M}$. We find immediately

$$F_{p_T} = \frac{\sigma_M}{\sigma^{\text{mix}}_M} - 1 = \sqrt{1 + \frac{\sigma^2_{\text{dyn}}}{\sigma^{2,\text{mix}}_M}} - 1. \tag{1.6}$$

Again, this is an exact relation. At RHIC $\sigma^2_{\text{dyn}} \ll \sigma^{2,\text{mix}}_M$, hence we can expand

$$F_{p_T} \simeq \frac{1}{2} \frac{\sigma^2_{\text{dyn}}}{\sigma^{2,\text{mix}}_M}, \tag{1.7}$$

which shows the proportionality of the two measures in the limit of small dynamical correlations.

A further simplification occurs when the distributions P_n are sharply peaked around some $\overline{n}$, which again is sufficiently well satisfied at RHIC. Then for a smooth function $f(n)$

$$\sum_n \frac{P_n}{n^z} f(n) \simeq \frac{f(\overline{n})}{\overline{n}^z}. \tag{1.8}$$

In this sharp limit

$$\begin{aligned} \sigma^2_{\text{dyn}} &\simeq \frac{1}{\overline{n}^2} \sum_{i\neq j} \text{cov}_{\overline{n}}(p_i, p_j), \\ \sigma^{2,\text{mix}}_M &\simeq \frac{\sigma^2_{\overline{n}}(p)}{\overline{n}}. \end{aligned} \tag{1.9}$$

For other measures the results are similar. For $\Sigma^2_{p_T}$ we have from definition $\Sigma^2_{p_T} \equiv \sigma^2_{\text{dyn}}/\overline{p}^2$, for Φ_{p_T} under conditions 1) and 2)

$$\Phi_{p_T} \equiv \sqrt{\frac{\sigma^2_S}{\overline{n}}} - \sigma_{\overline{n}}(p) \simeq \frac{\sum_{i\neq j} \text{cov}_{\overline{n}}}{2\overline{n}\sigma_{\overline{n}}(p)}, \tag{1.10}$$

where $S_n = p_1 + \ldots + p_n$.

Conclusion: Since at RHIC conditions 1) and 2) hold, the popular measures of event-by-event fluctuations are proportional to the covariance. Full information on correlations could be acquired by simply evaluating the covariance $\sum_{i\neq j} \text{cov}_n$ for each n. If 1) or 2) are relaxed, then, of course, the measures are no longer equivalent, but they are still related to the sum of the weighted covariances at various n in the way dependent on the particular measure.

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Finally, we make a digression concerning the *inclusive* distributions, not used in our derivations but appearing frequently in similar studies. They should not be confused with the marginal distributions, to which they are related as follows:

$$\rho_{\rm in}(p) = \sum_n P_n \int dp_1 \dots dp_n \sum_{i=1}^{n} \delta(p-p_i)\rho_n(p_1,...p_n),$$
$$\rho_{\rm in}(p,q) = \sum_n P_n \int dp_1 \dots dp_n \sum_{i\neq j=1}^{n} \delta(p-p_i)\delta(q-p_j)\rho_n(p_1,...p_n), \tag{1.11}$$

with the properties

$$\int dp\rho_{\rm in}(p) = \overline{n},$$
$$\int dpdq\rho_{\rm in}(p,q) = \overline{n(n-1)},$$
$$\overline{p_{\rm in}} = \sum_n nP_n\overline{p_n}. \tag{1.12}$$

1.1 Discussion

Staś Mrowsczynski (SM): What about zero multiplicity events? (divide by zero error in def of M). Better to involve measurements involving covariances

Tom Trainor (TT): n^2 not in the measurements. Claims his formula does not have σ^2_{dyn}

$$\frac{1}{n} = \frac{1}{(1+\overline{n}+\delta n)} = \frac{1}{\overline{n}}(1+\frac{\sigma_n^2}{\overline{n}^2}+) \tag{1.13}$$

WB: Limit to $n > 1$

TT: Certain measures have certain properties. Can design proxies for them, but that's not what we're dealing with (i.e. the covariances)

Marek Gazdzicki (MG): What are we designing it for? in order to make valid conclusions, have to measurement that behaves in intensive way, so insensitive to fine details and works for small mult. behaves sensibly for superposition model

WB: Claims his formula is same content as ϕ_{p_T} along with a small correction which is neglibible. Bets 30-year-old bottle of wine that will get same result as MG

2. Item 2: Tom Trainor

$+$ and $-$ vs. $+-$

2 combinations of scaled variances + scaled covariance

- isoscalar: $w^+ + w^- + 2w^\pm$ “charge independent” (CI)
- isovector: $w^+ + w^- - 2w^\pm$ “charge dependent” (CD)

The naming is to avoid "shock". Different correlations have different physical origins

- isoscalar: minijets

- isovector: net charge correlations

Linear combinations decouple. w+ and w- separately couple two different pieces of physics. There is an analog to polarization.

Roy Lacey (RL): If you look at like-sign, it's still jet correlations. The charge-dependent term just enhances correlation.

TT: You didn't notice a subtlety: CI shows peak and ridge, CD has hole at origin, even after a p_T cut, so this is jet physics. There are more unlike pairs in a jet than like sign

At high-z, charges balance, while at low-z, there's no need. As the multiplicity in a jet increases, then you start getting symmetry. In other words, a high-p_T jet has no CD, while a low-p_T jet does.

RL: How does one handle experimental situation? If you start to mix species, you begin to fold in more physics that makes situation complicated. In other words, the complications are in the inclusive measurements.

TT: Q,S,B (charge, strangeness, baryon number) are all in there.

Gorenstein (MG): Statistical model is good for total multiplicity, but experiment only has part. To introduce acceptance, you ignore cross correlations to start with. It creates complications when you included resonances that decay into both charges.

TT: We make linear combinations to decompose the physics, to evaluate the best way to spend research effort.

3. Item 3: Staś Mrowczynski

SM: I want to go home.

RL: Don't miss my talk tomorrow! [laughter]