PROCEEDINGS OF SCIENCE

PoS(NIC-IX)120

Shell Model Spin and Parity Dependent Nuclear Level Densities for Nuclear Reaction Rates

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We present a recently developed strategy [Nucl. Phys. **A 785C**, 142 (2005), Phys. Rev. C **69**, 041307(R) (2004), Phys. Rev. C **67**, 054309 (2003)] of calculating the spin and parity dependent shell model level density for application to astrophysics reaction rates. We also present an exact method for removing the spurious contribution to the nuclear level density in shell model spaces that are factorizable in center-of-mass (CoM) and intrinsic components.

International Symposium on Nuclear Astrophysics — Nuclei in the Cosmos — IX
June 25-30 2006
CERN, Geneva, Switzerland

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[*] Speaker.

[†] Support from the NSF grants PHY-0244453 and PHY-0555366 is acknowledged.

1. Introduction

Knowledge of reaction rates is essential for nuclear astrophysics. In many instances one cannot obtain experimental cross sections, therefore one has to rely on calculations. A widely used method to calculate cross sections for energies of interest in astrophysics is the Hauser-Feshbach method,[1] which requires exact knowledge of the spin and parity dependent nuclear level densities for excitation energies around the particle threshold.[2] We recently developed a strategy[3, 4, 5] of calculating the spin and parity dependent shell model level density for application to astrophysics reaction rates. The main ingredients are: (i) the extension of methods of the nuclear statistical spectroscopy[6] by exactly calculating the fixed spin first and second moments for different configurations, (ii) an exact decomposition of the space of many-body configurations in classes corresponding to different parities and number of harmonic oscillator excitations, (iii) developing new effective interactions for model spaces of interest starting from the G-matrix[10] and fixing/fitting monopole terms or/and linear combinations of two-body matrix elements to known experimental data, and (iv) an accurate estimate of the shell model ground state (g.s.) energy using the exponential convergence method (ECM).[7, 8, 9] We present spin and parity dependent nuclear level densities for cases of interest for the rp-process, such as ^{64}Zn. The calculations are done in the model space consisting of the $p_{3/2}$, $p_{1/2}$, $f_{5/2}$, and $g_{9/2}$ single particle (s.p.) orbits, which is less affected by the spurious contribution of the shell model center-of-mass excitations. An essential ingredient in these calculations is an effective interaction, which we obtained starting from a G-matrix theory[10] and is refined by a systematic fitting of the important linear combinations of two-body matrix elements to low-lying states in nuclei that are relevant for this model space.[11, 12]

2. Exact Removal of Spurious Contribution to the Nuclear Level Density

In several papers we developed a strategy, and the associated tools[3, 4, 5], to calculate the spin and parity dependent nuclear level densities as a function of the excitation energy using the methods of statistical spectroscopy. Other methods[13, 14, 15] calculate the density of states and later use several approximations to extract the nuclear level density, its spin and parity dependence. Our method requires the knowledge of the centroids and the widths for a restricted class of shell model configuration. The restriction to particular classes of configurations is necessary to be able to treat the harmonic oscillator $N\hbar\omega$ excitations, which are used to eliminate the contribution of the spurious (excited) center-of-mass (CoM) states. In Ref. [4] we showed how one can use this strategy to approximately eliminate the contribution of the spurious CoM states to the nuclear level density for low excitation energies. Here we describe a method of exact removal of the CoM spurious states form the nuclear level density for a certain class of $N\hbar\omega$ excitations (or combinations of them), if one knows the nuclear level density for all the states (including the spurious one) for several classes of $N'\hbar\omega$ excitations and their combinations. For a certain parity one needs to know the (nonspurious) level density for a given excitation energy E, a given total angular momentum J, and a combination of n, $(n+2)$, ..., $(n+2m)$ harmonic oscillator excitations, $\rho_{nsp}[E,J,(n)+(n+2)+...+(n+2m)]$ (in practice n=0 for the natural parity states and 1 for the unnatural parity states). Assuming that one knows the spurious densities for different J' and differ-

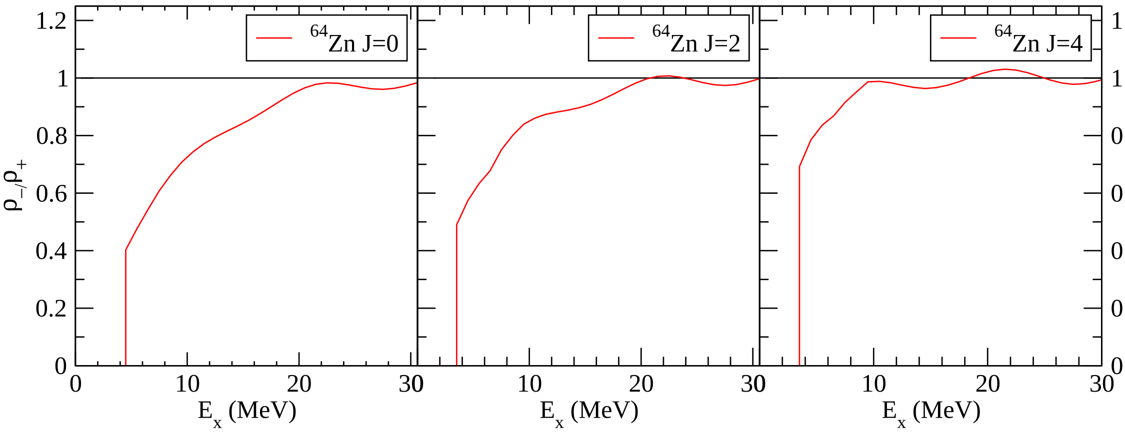

Figure 1: (Color online) Parity ratio of nuclear level densities for ^{64}Zn.

ent combinations of n, $(n+2)$, ..., $(n+2m)$ excitations, one can show that (details will be presented elsewhere[16])

$$\rho_{nsp}[E,J,(n)+(n+2)+...+(n+2m)] = \rho[E,J,(n)+(n+2)+...+(n+2m)] - \sum_{K=1}^{N} \sum_{J_K=J_{Kmin}}^{K,\ step\ 2} \sum_{J'=|J-J_K|}^{J+J_K} \rho_{nsp}[E,J',(n-K)+(n+2-K)+...+(n+2m-K)] \tag{2.1}$$

with the condition that if $(n-K)<0$ then

$$\rho_{nsp}[E,J',(n-K)+(n+2-K)...+(n+2m-K)] = \rho_{nsp}[E,J',(n+2-K)...+(n+2m-K)] \tag{2.2}$$

One necessary ingredient in the calculations is the knowledge of the energy of the ground states (g.s.) and of the yrast states of interest. The knowledge of the g.s. energy is necessary in order to identify the excitation energy of the system, The knowledge of the energies of the yrast states is necessary in order to find the thresholds in the spectrum for all the J's of interest. These energies are calculated either by direct diagonalization in the corresponding shell model space, or by using the exponential convergence method (ECM).[7, 8, 9] The yrast state energies are used in the expansion of the NLD in terms of finite range gaussians[3] to enforce NLD's correct threshold behavior. The effective interaction that are typically used in the $N\hbar\omega$ excitations shell model space can be obtained with the similarity transformation[17] or the G-matrix.[10]

3. Parity Ratio of Nuclear Level Densities

The methods developed in Refs. [3, 4, 5] and the techniques outlined in Section 2 can be used to address an outstanding problem, namely calculating the ratio of nuclear level densities of different parities for low excitation energies of interest in astrophysics. The standard approach based of the Fermi gas approximation to the nuclear level density makes the assumption that the contribution

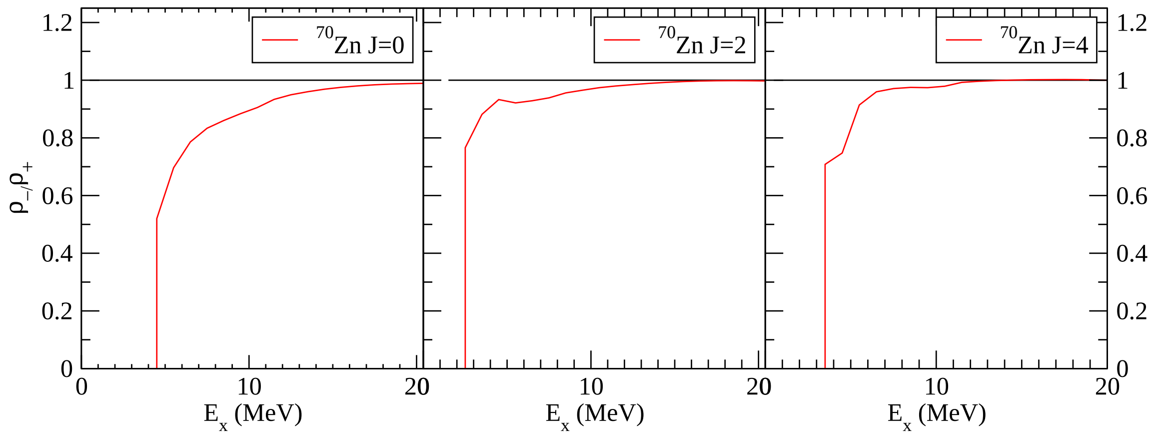

Figure 2: (Color online) Parity ratio of nuclear level densities for ^{70}Zn.

of both parities is the same.[2] While this approximation is reasonable for high excitation energies ($E > 20$ MeV) it is failing for excitation energies close to the particle separation that are of interest for nuclear astrophysics. An approximate solution to this problem was recently proposed[18] and some calculations are already available for nuclei around Zn.[19]

Here we present results of our methods for the Zn isotopes. We use a shell model space consisting of the $p_{3/2}$, $p_{1/2}$, $f_{5/2}$, and $g_{9/2}$ single particle orbits. This model space is less affected by the spurious contribution of the shell model center-of-mass excitations because there is no direct $\Delta J^{\pi} = 1^{-}$ coupling between any two orbits. The effective interaction used was developed starting with the G-matrix generated form the BonnC bare interaction, and by fitting certain linear combinations of matrix elements to about 600 levels in nuclei relevant for this model space. The results of the T=1 part of the interaction were presented in Ref. [11]. The full interaction will be described elsewhere.[12]

The results of the ratio of negative and positive level densities for $J = 0, 2, 4$ are presented in Fig. 1 for ^{64}Zn, and in Fig. 2 for ^{70}Zn. One can observe that: (i) the parity ratio of level densities significantly deviates from 1 for excitation energies smaller than 20 MeV, (ii) for each J-value there is a different threshold that can be accurately described with the present method.

4. Conclusions and Outlook

In conclusion, we further developed a strategy of calculating the spin and parity dependent nuclear level densities by proposing an exact method of removing the spurious center-of-mass contribution to the level density. We used our methodology to calculate the ratio of negative to positive level densities of different J's for two of the Zn isotopes. We showed that for low excitations energies of interest for nuclear astrophysics this ratio significantly deviates from one. In addition, this ratio has a threshold behavior that could be difficult to describe with other methods.

Our method does not require fixing parameters, other than the effective interaction that is usually done by starting with a G-matrix effective interaction and by further adjusting some combinations of matrix elements in order to describe some energy levels in a set of nuclei that can be described in the chosen model space. For lighter nuclei it might be possible to reliably use the effective interactions generated via the similarity transformation[17], but more studies are necessary to validate this approach. We plan to use these interactions to calculate the spin and parity dependent nuclear level densities in ^{26}Mg, a nucleus of interest for nuclear astrophysics.

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