NLO calculation of the $\Delta F = 2$ Hamiltonians in the MSSM and phenomenological analysis of the $B - \overline{B}$ mixing *

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We present the NLO corrections to the Wilson coefficients of the $\Delta F = 2$ Hamiltonians in the strong interacting sector of the MSSM, responsible for neutral meson oscillations. Such corrections, combined with the NLO anomalous dimension matrix for the $\Delta F = 2$ operators and the corresponding hadronic matrix elements calculated on the lattice, allow for the first time a full NLO phenomenological analysis of the observables related to meson oscillations. We present preliminary results in the $B_d - \overline{B}_d$ sector.

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1. Introduction

Observables related to meson oscillations represent among the best measured quantities in particle physics. This is true particularly for the $K - \overline{K}$ and the $B_d - \overline{B}_d$ cases, where experiments at K- and B-factories have pushed precisions to the permil level and new quantities in the D and B_s sectors promise to add up thanks to now-running and forthcoming experiments. Experimental commitment is related to the special interest meson oscillations have on the theoretical side: they constitute eminent sources of so-called flavor changing neutral currents (FCNC's) possibly accompanied by a sizeable violation of the CP symmetry. FCNC's and CP violation are in turn the effects that most naturally (and copiously) occur in basically all extensions of the SM, and in particular in SUSY, that, notwithstanding the success of the SM, represents the "standard way" beyond it [1]. The most popular low-energy realization of SUSY is provided by the MSSM, that however features 'by default' a huge space of parameters. Placing high precision constraints on the latter thus becomes of capital importance, considering that it can provide key handles on possibly ungrasped symmetries. To this aim, meson oscillations constitute privileged observables, recalling their experimental and theoretical 'virtues', as mentioned above.

The amplitudes for meson oscillations can be accessed in the Effective Hamiltonian (EH) approach, through the so-called $\Delta F = 2$ EH's, reading as

$$\mathcal{H}_{\text{eff}}^{\Delta F=2} = \sum_i C_i Q_i \,, \tag{1.1}$$

where the Q_i are local operators of increasing mass dimension and the C_i the corresponding 'couplings' (Wilson coefficients). In the MSSM and restricting to the $\Delta B = 2$ case ($B_d - \overline{B}_d$) a suitable basis for the Q_i up to dimension 6 is given by [2] ($P_{R,L} = (1 \pm \gamma_5)/2$)

$$
\begin{aligned}
Q_1 &= (\overline{\psi}_b^i \gamma_L^\mu \psi_d^i)(\overline{\psi}_b^j \gamma_{\mu L} \psi_d^j)\,, \quad Q_2 = (\overline{\psi}_b^i P_L \psi_d^i)(\overline{\psi}_b^j P_L \psi_d^j)\,, \quad Q_3 = (\overline{\psi}_b^i P_L \psi_d^j)(\overline{\psi}_b^j P_L \psi_d^i)\,, \\
Q_4 &= (\overline{\psi}_b^i P_L \psi_d^i)(\overline{\psi}_b^j P_R \psi_d^j)\,, \qquad Q_5 = (\overline{\psi}_b^i P_L \psi_d^j)(\overline{\psi}_b^j P_R \psi_d^i)\,,
\end{aligned}
\tag{1.2}
$$

plus $\tilde{Q}_{1,2,3}$ obtained from $Q_{1,2,3}$ with the substitution $L \rightarrow R$. In eq. (1.2) indices i, j denote color.

The C_i in eq. (1.1) are calculated by requiring that the amplitude $\mathscr{A}$ for meson oscillation, evaluated in the full (*i.e.* in the MSSM) and in the effective theory (1.1) be the same below a high-energy threshold M_{SUSY}. Since the C_i encode the short-distance 'part' of the process, the equality $\mathscr{A}_{\text{full}} = \mathscr{A}_{\text{eff}}$ can be imposed by choosing external quark states with arbitrary kinematics.

The $C_i(\mu \approx M_{\text{SUSY}})$ must then be evolved through RG flow to a scale $\mu \sim$ few GeV. The anomalous dimension for the operator basis (1.2) needed in the RG equations is known to NLO accuracy [3]. The amplitude for $B_d - \overline{B}_d$ mixing is finally obtained by multiplying the $C_i(\mu)$ by the matrix elements $\langle Q_i \rangle(\mu)$ between the physical states of interest, evaluated on the lattice [4].

The leading order (LO) C_i for $\Delta F = 2$ Hamiltonians in the strong sector of the MSSM were first calculated in refs. [2]. They are simply obtained by computing four gluino-squark 'box' diagrams (analogous to the W-quark boxes in the SM), and rewriting their amplitude in terms of tree-level matrix elements of the operators (1.2).

It is clear that knowledge of the next-to-leading order (NLO) corrections to the C_i is the missing ingredient for a full NLO analysis of $\Delta F = 2$ processes. Such corrections are the subject of the present contribution. In particular, section 2 briefly presents the highlights of the calculation, that

is dwelt on in a forthcoming publication. Section 3 addresses the phenomenological analysis, presenting preliminary results for the $B_d - \overline{B}_d$ system. A thorough analysis is again postponed to an upcoming paper.

2. NLO corrections to $\Delta F = 2$ Hamiltonians

NLO corrections to the C_i entering eq. (1.1) are calculated by enforcing the equality $\mathscr{A}_{\text{full}} = \mathscr{A}_{\text{eff}}$ at the scale M_{SUSY} up to two loops in the full theory and correspondingly one loop in the effective one. The two-loop diagrams mainly consist of gluon or squark corrections to the LO boxes: an inventory taking into account multiplicities and 'equivalences' results in around 70 diagrams. Hence, to be on the safe side as for the generation and the initial treatment of the full amplitude, the Mathematica package FeynArts [5] was an essential tool.

To two-loop order it becomes important to choose a regularization scheme for ultra-violet (UV) divergences as well as a kinematical configuration for the external quark legs. Concerning the first issue, we performed the whole calculation in both the NDR and DRED ($\overline{\text{MS}}$) schemes. As for external legs, we chose massless and zero-momentum quarks, since this leads to enormous simplifications in the algebra and in the treatment of the two-loop integrals, that in this way are just "vacuum" ones. This choice brings about infra-red (IR) divergences that however must cancel in the matching. In the intermediate steps IR divergences were coped with either by dimensional regularization (like UV ones) or by endowing gluon propagators with a mass term.

We then compared the calculations in all the resulting UV-IR regularization schemes and obtained perfect consistency, taking into account the findings of [6] for the treatment of evanescents and of [7] for the fact that NDR breaks SUSY. Other important checks of the calculated C_i were the already mentioned disappearance of IR divergences and the fact that the μ-dependence fulfills the relevant RG equations. Explicit formulae will be reported elsewhere.

We note that the final results are in the so-called mass insertion approximation (MIA) for the entries of the squark mass matrices in the 'super-CKM' basis: within the MIA diagonal entries are approximated with a common mass term M_s^2 appropriately chosen (1st approx.) and the off-diagonal ones Δ_{XY} (X,Y being the four possible chiralities) are considered small with respect to M_s^2 and expanded as mass insertions $\delta_{XY} = \Delta_{XY}/M_s^2 \ll 1$ (2nd approx.). The MIA enormously simplifies the phenomenological analysis and in our NLO case also decides for the *analytical* feasibility of the two-loop integrals.

3. Phenomenological analysis of the $B_d - \overline{B}_d$ system

The calculated NLO corrections to the C_i at the matching scale, evolved to the scale μ with the NLO ADM as reported in [3], permit for the first time a full NLO control on $\Delta F = 2$ Hamiltonians (1.1). This allows a correspondingly accurate analysis of meson-antimeson oscillations, updating the results of [8]. We restrict here to the $B_d - \overline{B}_d$ sector and use the findings of ref. [4] for the lattice determination of the matrix elements for the effective operators (1.2) in order to access the relevant amplitude $\mathscr{A}_{B_d} = \langle B_d | (\mathscr{H}_{\text{eff}}^{\Delta F = 2})^{\text{SM+SUSY}} | \overline{B}_d \rangle$.

Our analysis proceeds as follows: one extracts with normal distributions around their central experimental values all the (SM) parameters needed in $\mathscr{A}_{B_d}$ (CKM entries are taken from tree-level processes only, to leave SUSY unconstrained). The gluino mass is set to $M_{\tilde{g}} = 500$ GeV

and $x = M_{\tilde{g}}^2 / M_s^2 = 1$, while the mass insertions are extracted with flat distributions according to the intervals: $\text{Abs}(\delta_{XY}) \in [0, 1]$ and $\text{Arg}(\delta_{XY}) \in (-\pi, \pi]$. Therewith one has MonteCarlo determinations for $\mathscr{A}_{B_d}$, that are compared with experiments via the observables $\Delta M_d = 2\text{Abs}(\mathscr{A}_{B_d})$ and $2\beta_{\text{eff}} = \text{Arg}(\mathscr{A}_{B_d})$, the latter being accessed by studying the decays $B_d \to J/\psi K_S^{(*)}$. One can then place constraints on the mass insertions by 'switching on' one δ_{XY} at a time according to the choices: '*LL* only', '*RR* only', '*LL* = *RR*', '*LR* only', '*RL* only', '*LR* = *RL*'. Barring accidental cancellations among amplitudes proportional to different δ_{XY} parameters, the results can be interpreted as maximum ranges allowed by the present experimental information to the various mass insertions. Constraints on δ_{LL} are displayed in fig. 1, left panel, whereas the right panel nicely shows the reduction in the scale dependence of $\mathscr{A}_{B_d}$ when only *LL* and *RR* insertions are kept.

The 'hierarchical' pattern of the constraints, *e.g.* $\max(\delta_{LL}) \gg \max(\delta_{RL})$, represents an *a posteriori* justification for the MIA, wherein possibly large interference effects among squark mass matrix parameters are not taken into account. Such pattern also motivates our separate consideration of *LL*, *RR* and respectively *LR*, *RL* insertions in the analysis of the scale dependence.
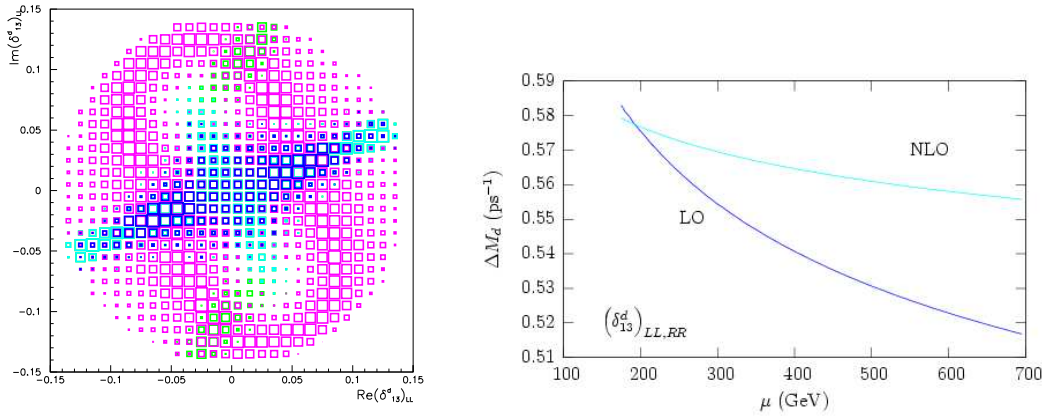

Figure 1: Left panel: allowed ranges for $\text{Re}(\delta_{LL}^{13})$ vs. $\text{Im}(\delta_{LL}^{13})$. The sides of the 'boxes' are proportional to the number of 'events' as weighted by the MonteCarlo. Constraints for different observables are superimposed with different colors: ΔM_d only (magenta), $\sin(2\beta)$ only (green), $\sin(2\beta)\,\&\,\cos(2\beta)$ (cyan), all the constraints (blue). Right panel: scale dependence of the *LL* and/or *RR* terms in the amplitude in the cases of Wilson coefficients to LO or to NLO accuracy (similar results hold in the *LR* and/or *RL* case).

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