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Quark-Lepton Complementarity with Renormalization Effects through Threshold Corrections

Sin Kyu Kang*
School of Physics, Seoul National University, Korea
E-mail: `skkang@phya.snu.ac.kr`

C. S. Kim
Department of Physics, Yonsei University, Korea
E-mail: `cskim@yonsei.ac.kr`

Jake Lee
Department of Physics, Yonsei University, Korea
E-mail: `jilee@cskim.yonsei.ac.kr`

The recent experimental measurements of the solar neutrino mixing angle θ_{sol} and the Cabibbo mixing angle θ_C reveal a surprising relation, $\theta_{sol}+\theta_C \simeq \frac{\pi}{4}$. While this empirical relation has been interpreted as a support of the idea of grand unification, it may be merely accidental in the sense that reproducing the relation at a low energy in the framework of grand unification may depend strongly on the renormalization effects whose size can vary with the choice of parameter space. We note that the lepton mixing matrix derived from quark-lepton unification can lead to a shift of the complementarity relation at low energy. While the renormalization group effects generally lead to additive contribution on top of the shift, we show that the threshold corrections which may exist in some intermediate scale new physics such as supersymmetric standard model can diminish it, so we can achieve the complementarity relation at a low energy.

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*Speaker.

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Recently, it has been noted that the solar neutrino mixing angle θ_{sol} and the Cabibbo angle θ_C reveal a surprising relation

$$\theta_{sol} + \theta_C \simeq \frac{\pi}{4}, \tag{1}$$

which is satisfied by the experimental results $\theta_{sol} + \theta_C = 45.4^\circ \pm 1.7^\circ$ to within a few percent accuracy [1]. This quark-lepton complementarity (QLC) relation (1) has been interpreted as an evidence for quark-lepton unification [2]. Yet, it can be a coincidence in the sense that reproducing the exact QLC relation (1) at low energy scale in the framework of grand unification depends on the renormalization effects whose size can vary with the choice of parameter space [3].

A parametrization of the PMNS mixing matrix in terms of a small parameter whose magnitude can be interestingly around $\sin\theta_C$ has been proposed as follows [4]:

$$U_{\rm PMNS} = U^\dagger(\lambda) U_{\rm bimax} \,. \tag{2}$$

Here $U(\lambda)$ is a mixing matrix parameterized in terms of a small parameter λ and $U_{\rm bimax}$ corresponds to the bi-maximal mixing matrix [5]. In this work, we show that the lepton mixing matrix given in the form of Eq. (2) with $U(\lambda) \sim U_{\rm CKM}$ can be indeed realized in the framework of grand unification with symmetric Yukawa matrices when we incorporate seesaw mechanism, and examine whether or not $U_{\rm PMNS}$ given by (2) can predict the QLC relation (1) exactly.

It is necessary to take into account the renormalization effects on $U_{\rm PMNS}$ when one compares the prediction at a high energy scale with the QLC relation observed at low energy scale [6]. In MSSM with large $\tan\beta$ and the quasi-degenerate neutrino mass spectrum, the RG effects are generally large and can enhance the mixing angle θ_{12} at low energy [6]. Such an enhancement of θ_{12} is not suitable for achieving the QLC relation (1) at low energy . In this work, we show that the sizeable *threshold corrections* which may exist in the MSSM [7, 8] can diminish the deviation from the QLC relation while keeping θ_{23} almost maximal and θ_{13} small, so that the QLC relation at low energy can be achieved when the RG effects are suppressed.

The quark Yukawa matrices Y_u, Y_d are given by $Y_u = U_u Y_u^{diag} V_u^\dagger$, $Y_d = U_d Y_d^{diag} V_d^\dagger$, from which CKM mixing matrix is given by $U_{\rm CKM} = U_u^\dagger U_d$. The charged lepton Yukawa matrix is given by $Y_l = U_l Y_l^{diag} V_l^\dagger$. For the neutrino sector, we introduce one right-handed singlet neutrino per family which leads to the seesaw mechanism, according to which the light neutrino mass matrix is given by $M_\nu = \left(U_0 M^{diag}_{\rm Dirac} V_0^\dagger\right) \frac{1}{M_R} \left(V_0^* M^{diag}_{\rm Dirac} U_0^T\right)$, where U_0 and V_0 are the left-handed and right-handed mixing matrices of $M_{\rm Dirac}$, respectively. We can then rewrite M_ν as follows $M_\nu = U_0 V_M M_\nu^{diag} V_M^T U_0^T$, where V_M represents the rotation of $M^{diag}_{\rm Dirac} V_0^\dagger \frac{1}{M_R} V_0^* M^{diag}_{\rm Dirac}$. Then, $U_{\rm PMNS}$ is given by

$$U_{\rm PMNS} = U_l^\dagger U_\nu = U_l^\dagger U_0 V_M \,. \tag{3}$$

Now, let us consider how $U_{\rm PMNS}$ given by Eq. (3) can be related with $U_{\rm CKM}$ in the quark-lepton unification.

(A) Minimal quark-lepton unification : Since the down-type quarks and the charged leptons are in general assigned into a multiplet in grand unification, we can assume that $Y_e = Y_d^T$, $\quad Y_u = Y_u^T$. Then, we deduce that $U_l = V_d^*$ from which $U_{\rm PMNS} = V_d^T U_0 V_M$. From this expression for $U_{\rm PMNS}$, we see that the contribution of $U_{\rm CKM}$ may appear in $U_{\rm PMNS}$ when $Y_\nu = Y_u$ which can be realized in

$SO(10)$. Then, one can obtain $U_{\rm PMNS} = V_d^T U_d U_{\rm CKM}^\dagger V_M$ and then requiring symmetric form of Y_d, we finally obtain

$$U_{\rm PMNS} = U_{\rm CKM}^\dagger V_M, \tag{4}$$

where V_M has bi-maximal mixing pattern. In this way, $U_{\rm PMNS}$ can be connected with $U_{\rm CKM}$. To see whether the parametrization of $U_{\rm PMNS}$ given by (4) can lead to the QLC relation (1), it is convenient to present $U_{\rm PMNS}$ for the CP-conserving case as follows:

$$U_{\rm PMNS} = U_{\rm CKM}^\dagger U_{23}^m U_{12}^m \equiv U_{23}(\theta_{23}) U_{13}(\theta_{13}) U_{12}(\tfrac{\pi}{4} - \theta_{12}) \;, \tag{5}$$

where U_{12}^m and U_{23}^m correspond to the maximal mixing between (1,2) and (2,3) generations, respectively. The solar neutrino mixing $\sin\theta_{sol}$ then becomes $\sin\theta_{sol} \simeq \sin\left(\frac{\pi}{4} - \theta_C\right) + \frac{\lambda}{2}(\sqrt{2}-1)$. Thus, we see that the neutrino mixing matrix (5) originating from the quark-lepton unification obviously leads to a shift of the relation (1). Numerically, the shift amounts to $\delta\theta_{sol} \simeq 3^\circ$ and we can expect that renormalization effects on Eq. (5) may fill the gap between the QLC relation and the prediction for $\sin\theta_{sol}$ from high energy mixing matrix.

(B) Realistic quark-lepton unification : Although the minimal quark-lepton unification can lead to an elegant relation between $U_{\rm PMNS}$ and $U_{\rm CKM}$ as shown above, it indicates undesirable mass relations between quarks and leptons at the GUT scale such as $m_d^{diag} = m_l^{diag}$. Recently, a desirable form of $U_l^\dagger U_0$ has been suggested based on a well known empirical relation $|V_{us}| \simeq \sqrt{\frac{m_d}{m_s}} \simeq 3\sqrt{\frac{m_e}{m_\mu}}$ [9], from which $\sin\theta_{sol}$ is given by $\sin\theta_{sol} \simeq \sin\left(\frac{\pi}{4} - \theta_C\right) + \frac{\lambda}{2}\left(\sqrt{2} - \frac{1}{3}\right)$. Numerically, the deviation from the QLC relation amounts to $\delta\theta_{sol} \simeq 7^\circ$. We consider a possibility that the threshold corrections can diminish the deviation from the QLC relation.

Now, let us examine how the renormalization effects can diminish the deviation from the QLC relation. In general, the radiative corrections to the effective neutrino mass matrix are given by:

$$M_\nu = I \cdot M_\nu^0 \cdot I = I \cdot U_{\rm CKM}^T U_{23}^{m^*} M_{D12} U_{23}^{m\dagger} U_{\rm CKM} \cdot I \;, \tag{6}$$

where $M_D = Diag[m_1, m_2, m_3]$, $M_{D12} = U_{12}^{m^*} M_D U_{12}^{m\dagger}$, and the matrix $I \equiv I_A \delta_{AB}, (A, B = e, \mu, \tau)$ stands for the radiative corrections. The correction I generally consists of two parts $I = I^{RG} + I^{TH}$ where I^{RG} are RG corrections and I^{TH} are electroweak scale threshold corrections [7]. The typical size of RG corrections I^{RG} is known to be about 10^{-6} in the SM and MSSM with small $\tan\beta$, and thus negligible. In addition, supersymmetry can induce flavor dependent threshold corrections related with slepton mass splitting which can dominate over the charged lepton Yukawa corrections [8]. We have numerically checked that RG evolution from the seesaw scale to the weak scale *enhances* the size of θ_{12} in the case that θ_{13} and θ_{23} are kept to be small and almost maximal mixing, respectively. Thus, the case of sizable RG effects is not suitable for our purpose. Instead, we examine whether the threshold corrections can be suitable for diminishing the deviation from the QLC relation while keeping θ_{23} nearly maximal and θ_{13} small in the case that RG effect is negligible. To achieve our goal, we note that the contribution I_e should be dominant over $I_{\mu,\tau}$ because only I_e can lead to the right amount of the shift of θ_{12} while keeping the changes of θ_{23} and θ_{13} small. Taking $|I_e| >> |I_{\mu,\tau}|$, the neutrino mass matrix corrected by the leading contributions is rewritten as follows:

$$M_\nu \;\simeq\; U_{\rm CKM}^T U_{23}^{m^*} \left[I_D + I_e \Lambda_\lambda\right] M_{D12} \left[I_D + I_e \Lambda_\lambda^\dagger\right] U_{23}^{m\dagger} U_{\rm CKM} \;, \tag{7}$$

	m_1 (eV)	0.15	0.1	0.05
I_e	(A)	-4.0×10^{-5}	-8.5×10^{-5}	-3.4×10^{-4}
I_e	(B)	-1.0×10^{-4}	-2.2×10^{-4}	-8.6×10^{-4}

Table 1: Parameter set (I_e, m_1) leading to the QLC relation. The rows (A) and (B) correspond to the minimal unification and realistic case, respectively.

where I_D is 3×3 identity matrix, and the matrix Λ_λ is given in terms of λ.

To see how much the lepton mixing angles can be shifted by I_e, we do numerical analysis in a model independent way based on the form given by Eq. (7), and by taking $\Delta m^2_{sol} \equiv m_2^2 - m_1^2 \simeq 7.1\times10^{-5}$ eV^2 and $\Delta m^2_{atm} \equiv m_3^2 - m_2^2 \simeq 2.5\times10^{-3}$ eV^2. Varying the parameter I_e and the smallest light neutrino mass m_1, we find which parameter set (I_e, m_1) can lead to the QLC relation exactly and the results are presented in Table I. The first and the second row in Table I correspond to the minimal unification and realistic case, respectively. In our analysis, we have also checked that θ_{23} is almost unchanged, whereas the shift of θ_{13} is about 1° for both cases (A) and (B). From Table I, we see that a larger value of I_e is required to achieve the relation (1) as m_1 goes down.

To achieve $|I_e| >> |I_{\mu,\tau}|$, we can consider a dominant contribution of chargino (pure W-ino) to I_e in MSSM, and it turns out that the size of $|I_e|$ is about 10 times larger than that of $|I_{\mu,\tau}|$ for $M_e \sim 2M_{\mu,\tau}$, and the value of I_e becomes negative and of the order of $10^{-4} \sim 10^{-3}$ for $x_e \gtrsim 0.65$, which are required to achieve the exact QLC relation at low energy.

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