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Why do a precision measurement of $\delta m^2_{\rm atm}$ in the ν_e ($\bar{\nu}_e$) disappearance channel?

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We discuss why high precision measurements of $\delta m^2_{\rm atm}$ in the $\nu_e/\bar{\nu}_e$ disappearance channels would be desirable in conjuction with the $\delta m^2_{\rm atm}$ high precision measurements that will be performed in the ν_μ and $\bar{\nu}_\mu$ disappearance channels by long baseline experiments such as T2K and NOνA. We show that if these measurements can achieve the challenging precision of about 0.5%, it will be possible to determine the mass hierarchy of the neutrino sector without the need of matter effects.

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1. Going from two to three neutrinos description

The simplest and most widely accepted extension of the Standard Model of particle physics which explains the neutrino flavor transitions that have been observed so far in atmospheric, solar, reactor and accelerator neutrino experiments [1], is to allow neutrinos to have mass and mixings, similar to the quark sector. In this case, flavor transitions can be accounted for by neutrino oscillations.

Up to now the experimental data can be well understood in terms of neutrino oscillation between two generations. The results from the latest KamLAND [2] experiment combined with solar neutrino experiments can constrain the allowed range of the solar mass squared difference [3]. Maximal mixing, $\sin^2\theta_{12} = 0.5$, has been ruled out at greater than 5 σ and the solar neutrino data is consistent with $\nu_e \to \nu_\mu$ and/or ν_τ. On the other hand, the atmospheric neutrino data from SuperKamiokande [4] combined with the latest results from the K2K long baseline experiment can constrain the range of allowed values for the atmospheric mass squared difference, δm^2_{32} and the mixing angle, θ_{23}. The atmospheric data is consistent with $\nu_\mu \to \nu_\tau$ oscillations but the sign of δm^2_{32} is unknown. This sign is positive (negative) if the doublet of neutrino mass eigenstates, 1 and 2, which are responsible for the solar neutrino oscillations have a smaller (larger) mass than the 3rd mass eigenstate. This is the mass hierarchy question. Including also the constraint on the involvement of the ν_e at the atmospheric δm^2 coming from the Chooz reactor experiment we can summarize our currently knowledge as: $+7.3\times 10^{-5}\text{eV}^2 < \delta m^2_{21} < +9.0\times 10^{-5}\text{eV}^2$, $0.25 < \sin^2\theta_{12} < 0.37$, $1.5\times 10^{-3}\text{eV}^2 < |\delta m^2_{32}| < 3.4\times 10^{-3}\text{eV}^2$, $0.36 < \sin^2\theta_{23} \le 0.64$ and $0 \le \sin^2\theta_{13} < 0.04$ at the 90% confidence level. For a review of all these limits see Ref. [1].

So far the inclusion of genuine three flavor effects has not been important because these effects are controlled by the two small parameters

$$\frac{\delta m^2_{21}}{\delta m^2_{32}} \approx 0.03 \quad \text{and/or} \quad \sin^2\theta_{13} \le 0.04. \tag{1.1}$$

However as the accuracy of the neutrino data improves it will become inevitable to take into account genuine three flavor effects including CP and T violation.

Genuine three generation effects make the effective atmospheric neutrino δm^2 measured by disappearance experiences, in principle, flavor dependent even in vacuum and thus sensitive to the mass hierarchy and even to the CP phase. This observation suggests an alternative way to access the mass hierarchy by comparing precisely measured values for the atmospheric δm^2 in $\bar{\nu}_e \to \bar{\nu}_e$ (reactor) and $\nu_\mu \to \nu_\mu$ (accelerator) modes. Here we briefly discuss this rather interesting but experimentally challenging possibility, for more details see Ref. [5].

2. Going from three to two neutrinos description

Assuming three active neutrinos only, it can be shown [5] that if one defines $\delta m^2_{\rm eff}|_\alpha$, the effective atmospheric mass squared difference for the α-flavor as

$$\delta m^2_{\rm eff}|_\alpha \equiv \frac{|U_{\alpha 1}|^2\ \delta m^2_{31} + |U_{\alpha 2}|^2\ \delta m^2_{32}}{|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2}, \tag{2.1}$$

then the survival probability for the α-flavor neutrino, in vacuum, can be written as

$$1 - P(\nu_\alpha \to \nu_\alpha) = \quad 4|U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2)\sin^2\Delta_{\rm eff}|_\alpha + \mathscr{O}(\Delta^2_{21}), \tag{2.2}$$

where $\Delta_{\rm eff}|_\alpha = \delta m^2_{\rm eff}|_\alpha L/4E$, $\Delta_{ij} = \delta m^2_{ij}L/4E$, $\delta m^2_{ij} = m_i^2 - m_j^2$ and $U_{\alpha i}$ are elements of the MNS mixing matrix. The three Δ_{ij} are not independent since the δm^2_{ij}'s satisfy the constraint, $\delta m^2_{31} = \delta m^2_{32} + \delta m^2_{21}$. This approximate solution is excellent provided that $\Delta_{21} \ll 1$.

To understand the physical meaning of the effective atmospheric δm^2 it is useful to write it as follows

$$\delta m^2_{\rm eff}|_\alpha \ = m_3^2 - \langle m^2_\alpha\rangle_{12}, \qquad \text{where} \qquad \langle m^2_\alpha\rangle_{12} \equiv \frac{|U_{\alpha 2}|^2 m_2^2 + |U_{\alpha 1}|^2 m_1^2}{|U_{\alpha 1}|^2+|U_{\alpha 2}|^2}. \tag{2.3}$$

Now $\langle m^2_\alpha\rangle_{12}$ has a clear interpretation, it is the α-flavor weighted average mass square of neutrino states 1 and 2. Thus the effective atmospheric δm^2 is the difference in the mass squared of the state 3 and this flavor average mass square of states 1 and 2 and is clearly flavor dependent. It is now obvious that ν_e and ν_μ disappearance experiments measure *different* $\delta m^2_{\rm eff}$'s.

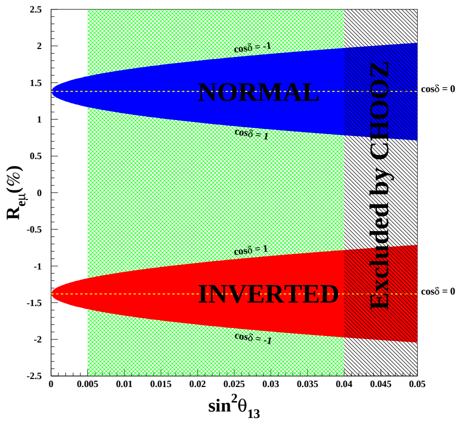

Fig. 1: Graph of $\Delta_{e\mu} \equiv (|\delta m^2_{\rm eff}|_e - |\delta m^2_{\rm eff}|_\mu)/|\delta m^2_{\rm eff}|$ as a function of $\sin^2\theta_{13}$ for the normal and inverted hierarchies showing the dependence on $\cos\delta$.

In Fig. 1, we show this difference normalized to the atmospheric δm^2 for the normal and inverted hierarchy, as a function of $\sin^2\theta_{13}$. For the normal hierarchy, independently of δ, this normalized ratio is always positive, while for the inverted one, it is always negative. While the size of difference between the two hierarchies is smallest for $\cos\delta = 1$, for this value of δ, the difference between the two hierarchies increases as $\sin^2\theta_{13}$ goes to zero, as can be seen from Eq.(2.4).

Whether the absolute value of $\delta m^2_{\rm eff}|_e$ is larger or smaller than the absolute value of $\delta m^2_{\rm eff}|_\mu$ depends on whether $|\delta m^2_{31}|$ is larger or smaller than $|\delta m^2_{32}|$. The relative magnitude of these two δm^2 is determined by whether the mass squared of the 3-state is larger or smaller than the mass squared of the 1- and 2-states, i.e. by the neutrino mass hierarchy. It is easy to show that the difference in the absolute value of the e-flavor and μ-flavor $\delta m^2_{\rm eff}$'s is given by

$$|\delta m^2_{\rm eff}|_e - |\delta m^2_{\rm eff}|_\mu = \pm\delta m^2_{21}(\cos 2\theta_{12} - \cos\delta\sin\theta_{13}\sin 2\theta_{12}\tan\theta_{23}), \tag{2.4}$$

where the $+$ sign ($-$ sign) is for the normal (inverted) hierarchy. Thus by precision measurements of both of these $\delta m^2_{\rm eff}$ one can determine the hierarchy and possibly even $\cos\delta$.

3. Final Discussion

We have calculated the required precision as function of the C.L. assuming that the two experiments have same % precision. In Fig. 2 the error is given as a fraction of the expected separation given by Fig. 1. Since the expected separation is approximately 1% we see that for a 90% C.L. determination of the hierarchy one would require $\sim$0.5% precision on *both* δm^2_{eff} measurements. Achieving such precision will require significant innovation.

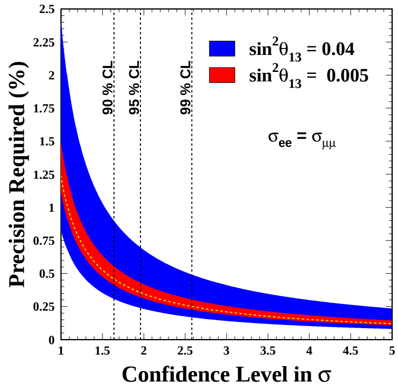

Fig. 2: The confidence level determination of the mass hierarchy verses the measurement error, σ, as a fraction of the expected separation between $|\delta m^2_{\rm eff}|_e$ and $|\delta m^2_{\rm eff}|_\mu$.

So high precision measurements of the effective atmospheric δm^2 in both the $\bar{\nu}_e \to \bar{\nu}_e$ and $\nu_\mu \to \nu_\mu$ channels can determine the neutrino mass hierarchy independent of matter effects. The sign of the difference determines the hierarchy. For any reasonable confidence level determination the precision required in *both* channels is a very challenging fraction of 1%. The next generation of long baseline experiments such as T2K and NOνA estimate their precision on the effective atmospheric δm^2 at 2%. However, so far there has been no physics reason to push this to a precision measurement. For the reactor channel the emphasis so far has been on the observation of non-zero θ_{13}, very little effort has been made on a precision determination of the effective atmospheric δm^2. This kind of precision, can perhaps be achieved in beta beam facility. We realize that to make these measurements to the precision suggested is very challenging experimentally however we encourage our experimental colleagues to give this some thought especially since this method is practically independent of the CP violating phase whereas the determination of the mass hierarchy in long baseline experiments is very difficult for some values of this phase.

References

[1] See M. Maltoni contribution in this proceedings for a review of these results.

[2] See J.-S. Ricol contribution in this proceedings.

[3] See S. Oser contribution in this proceedings.

[4] See M. Vagins contribution in this proceedings.

[5] H. Nunokawa, S. Parke and R. Zukanovich Funchal, Phys. Rev. D **72**, 013009 (2005) [hep-ph/0503283].