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Understanding the U(1) problem through dyon configuration in the Abelian projection

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We give a short review of the recently obtained result that the magnetic monopole promoted to the dyon due to the vacuum angle θ resolves the U(1) problem in the sense that the dyon obtained in this way gives a dominant contribution to the topological susceptibility [1].

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1. Introduction

It is very interesting and challenging to study a mechanism of the non-perturvative phenomena of QCD, such as quark confinement, chiral symmetry breaking, strong CP violation, etc. These non-perturbative phenomena are believed to be well understood in the unified way by considering the topologically nontrivial configurations of the gluon field. The U(1) problem or η meson problem [2] is also one of such phenomena. 't Hooft [3] pointed out that topologically nontrivial configurations such as instantons give the nonzero anomaly and suggested that instantons are the relevant topological objects related to the resolution of the U(1) problem[4]. However, it was not clear how to compute the η' mass. Moreover, it was pointed out that the Ward-Takahashi identity for the $U_A(1)$ current with the anomalous term contradicts with the quark–antiquark condensation in the instanton θ vacuum [5].

In this talk, we review our recent result that the U(1) problem is understood through the dyon configuration. A strategy for solving the U(1) problem along this line has already been discussed by Ezawa and Iwazaki [6] based on the idea of the Abelian projection proposed by 't Hooft [7]. However, they assumed in their analyses the *Abelian dominance* from the beginning and used an Abelian-projected effective theory which is conjectured to be derived from the Yang-Mills theory in the long distance. In contrast, extending the method developped by one of author[8], *we derive the Abelian-projected effective theory* based on the functional integration of the off-diagonal degrees of freedom from the Yang-Mills theory with the θ angle. We summarize a flowchart of our strategy in Figure 1.

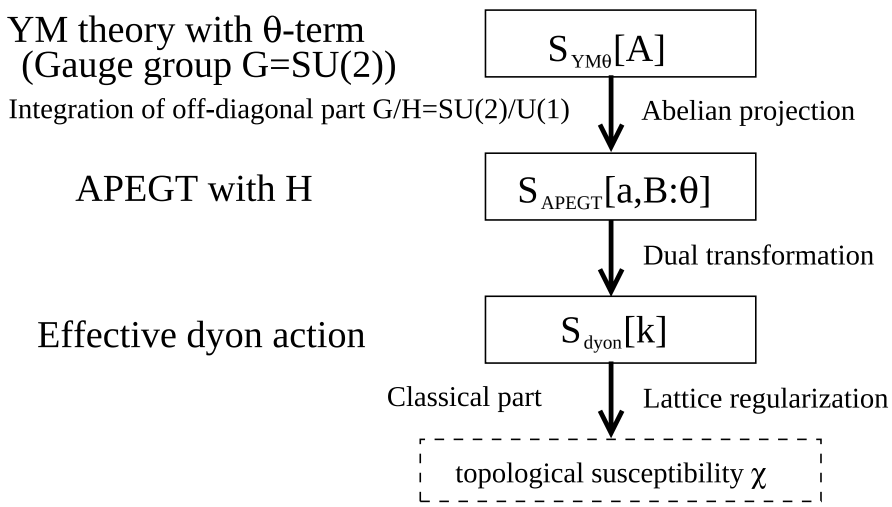

Figure 1: A flowchart of our strategy

2. Derivation of effective dyon action

We start from the SU(2) Yang-Mills (YM) action $S_{YM}[A]$ with the θ term $S_\theta[A]$:

$$S_{YM\theta}[A] = S_{YM}[A] + S_\theta[A], \tag{2.1}$$

$$S_{YM}[A] = -\frac{1}{2g^2}\int d^4x \mathrm{tr}(F_{\mu\nu}F^{\mu\nu}), \tag{2.2}$$

$$S_{\theta}[A] = \frac{\theta}{16\pi^2}\int d^4x \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}), \tag{2.3}$$

where the field strength $F_{\mu\nu}$ is defined by

$$F_{\mu\nu}(x) = \sum_{A=1}^{3} F^A_{\mu\nu}(x)T^A = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - ig[A_\mu(x), A_\nu(x)], \tag{2.4}$$

and $T^A (A = 1,2,3)$ is the generator of the Lie algebra of the gauge group $SU(2)$. The Hodge dual $\tilde{F}_{\mu\nu}$ of $F_{\mu\nu}$ is defined by

$$\tilde{F}_{\mu\nu}(x) \equiv \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}(x). \tag{2.5}$$

2.1 Step 1: Cartan decomposition

We decompose A_μ into the diagonal U(1) and the off-diagonal SU(2)/U(1) parts as

$$A_\mu(x) = a_\mu(x)T^3 + \sum_{a=1}^{2} A^a_\mu(x)T^a \in H \oplus (G-H). \tag{2.6}$$

where $a_\mu(x)$ and $A^a_\mu(x)$ are diagonal, off-diagonal gluon field, respectively. Accordingly, the field strength $F_{\mu\nu}$ is decomposed as

$$F_{\mu\nu} = [f_{\mu\nu}(x) + C_{\mu\nu}(x)]T^3 + S^a_{\mu\nu}(x)T^a, \tag{2.7}$$

$$f_{\mu\nu}(x) \equiv \partial_\mu a_\nu(x) - \partial_\nu a_\mu(x), \tag{2.8}$$

$$S^a_{\mu\nu}(x) \equiv D_\mu[a]^{ab}A^b_\nu(x) - D_\nu[a]^{ab}A^b_\mu(x), \tag{2.9}$$

$$C_{\mu\nu}(x)T^3 \equiv -i[A_\mu(x), A_\nu(x)], \tag{2.10}$$

where the covariant derivative $D_\mu[a]$ is defined by

$$D_\mu[a] = \partial_\mu + i[a_\mu T^3, \cdot], \quad D_\mu[a]^{ab} = \partial_\mu\delta^{ab} - \varepsilon^{ab3}a_\mu. \tag{2.11}$$

Next, the total action $S_{YM\theta}[A]$ is decomposed as

$$S_{YM\theta}[A] = \int d^4x\left\{-\frac{1}{4g^2}f_{\mu\nu}f^{\mu\nu} + \frac{\theta}{32\pi^2}f_{\mu\nu}\tilde{f}^{\mu\nu} - \frac{1}{4}g^2 B_{\mu\nu}B^{\mu\nu} + \frac{1}{2g^2}A^{\mu a}Q^{ab}_{\mu\nu}A^{\nu b}\right\}, \tag{2.12}$$

$$Q^{ab}_{\mu\nu} \equiv (D_\rho[a]D_\rho[a])^{ab}\delta_{\mu\nu} - 2\varepsilon^{ab3}f_{\mu\nu} + g^2 c_1\varepsilon^{ab3}\tilde{B}_{\mu\nu} + g^2 c_0\varepsilon^{ab3}B_{\mu\nu} - D_\mu[a]^{ac}D_\nu[a]^{cb}, \tag{2.13}$$

where we have introduced the auxiliary (antisymmetric tensor) field $B_{\mu\nu}$ according to

$$B_{\mu\nu} = g^{-2}(c_0 C_{\mu\nu} + c_1\tilde{C}_{\mu\nu}), \tag{2.14}$$

and two coefficients, c_0 and c_1 are determined as

$$c_0 = \sqrt{\frac{1}{2}\left(-1+\sqrt{1+\left(\frac{g^2\theta}{8\pi^2}\right)^2}\right)}, \quad c_1 = \sqrt{\frac{1}{2}\left(1+\sqrt{1+\left(\frac{g^2\theta}{8\pi^2}\right)^2}\right)} \tag{2.15}$$

to be equivalent to the original action $S_{YM\theta}[A]$.

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2.2 Step 2: Maximaly abelian gauge fixing

We perform the abelian projection in terms of BRST formalism and consider a gauge fixing for the off-diagonal part;

$$F^{\pm}[A,a] \equiv (\partial^{\mu} \pm i\xi a^{\mu})A^{\pm}_{\mu} = 0, \tag{2.16}$$

where we have used the $(\pm,3)$ basis, $O^{\pm} \equiv (O^1 \pm iO^2)/\sqrt{2}$. Here the gauge parameter $\xi = 0$ corresponds to the Lorentz gauge and $\xi = 1$ to (the differential form of) the maximal abelian gauge (MAG). In the BRST quantization, the GF condition (2.16) amounts to adding the following GF term and the Faddeev–Popov (FP) term [8],

$$\mathscr{L}_{GF+FP} = \phi^a F^a[A,a] + \frac{\alpha}{2}(\phi^a)^2 + i\bar{c}^a D^{\mu ab}[a]^{\xi} D^{bc}_{\mu}[a]c^c - i\xi\bar{c}^a[A^a_{\mu}A^{\mu b} - A^c_{\mu}A^{\mu c}\delta^{ab}]c^b, \tag{2.17}$$

where ϕ stands for the Langange multiplier field and $F^a[A,a]$ is a gauge fixing function defined by

$$F^a[A,a] = (\partial^{\mu}\delta^{ab} - \xi\varepsilon^{ab3}a^{\mu})A^b_{\mu} = D^{\mu ab}[a]^{\xi}A^b_{\mu}. \tag{2.18}$$

Thus the total Lagrangian is obtained by adding (2.17) to (2.12),

$$\mathscr{L} = \mathscr{L}_{YM\theta}[A] + \mathscr{L}_{GF+FP}. \tag{2.19}$$

We choose gauge parameters $\alpha = \xi = 1$ below.

2.3 Step 3: APEGT with H

We integrate out the off-diagonal fields, ϕ^a, A^a_{μ}, c^a, $\bar{c}^a$ belonging to the Lie algebla of SU(2)/U(1) and obtain the Abelian-projected effective gauge theory (APEGT) written in terms of the diagonal fields, a_{μ} and $B_{\mu\nu}$ *. As a result, we obtain APEGT †;

$$\begin{aligned} S_{APEGT}[a,B:\theta] = \int d^4x \Big[& -\frac{1+z_a-z'_a}{4g^2} f_{\mu\nu}f^{\mu\nu} - \frac{1+z_b}{4}g^2 B_{\mu\nu}B^{\mu\nu} \\ & -\frac{1}{2}z_c B_{\mu\nu}\tilde{f}^{\mu\nu} + \frac{\theta}{32\pi^2} f_{\mu\nu}\tilde{f}^{\mu\nu} + \frac{1}{2}z_d f_{\mu\nu}B^{\mu\nu} + \frac{1}{2}z_e B_{\mu\nu}\tilde{B}^{\mu\nu} \\ & +(\text{4-ghost terms}) + (\text{higher derivative terms})\Big], \end{aligned} \tag{2.20}$$

where z's are renormalization constants;

$$\begin{aligned} z_a &= -\frac{10}{3}\kappa\frac{g^2}{16\pi^2}\ln\mu^2, \quad z'_a = \frac{1}{3}\kappa\frac{g^2}{16\pi^2}\ln\mu^2, \quad z_b = \kappa\frac{g^2}{16\pi^2}\ln\mu^2, \\ z_c &= 2\kappa c_1\frac{g^2}{16\pi^2}\ln\mu^2, \quad z_d = 2\kappa c_0\frac{g^2}{16\pi^2}\ln\mu^2, \quad z_e = -\kappa\frac{g^4}{16\pi^2}\cdot\frac{g^2\theta}{16\pi^2}\ln\mu^2. \end{aligned} \tag{2.21}$$

We have introduced the second Casimir operator κ which is given for $G = SU(2)$ by

$$\kappa \equiv C_2(G) = \varepsilon^{3ab}\varepsilon^{3ab} = 2. \tag{2.22}$$

* See [1] for the detail of the calculations.

† We have neglected the ghost self-interaction terms and higher derivative terms.

2.4 Step 4 : Effective dyon action

The magnetic monopole current k_μ is defined by

$$k^\mu \equiv \partial_\nu \tilde{f}^{\mu\nu}, \quad \tilde{f}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} f_{\rho\sigma}. \tag{2.23}$$

Introducing the monopole currents in the APEGT (2.20) and integrating out of all fields except for k_μ, we obtain

$$S_{dyon}[k] = \int d^4x \frac{1}{2} g_m^2[\theta] k^\mu(x) D_{\mu\nu} k^\nu(x), \tag{2.24}$$

where $g_m[\theta] := g|\tau| = \sqrt{g_m^2 + q_m^2}$ ($g_m \equiv 4\pi/g, \quad q_m \equiv g\theta/2\pi$) and the kernel $D_{\mu\nu}$ stands for the massless vector propagator, e.g., $D_{\mu\nu} = (1/\partial^2)(\delta_{\mu\nu} - \partial_\mu \partial_\nu/\partial^2)$ in the Landau gauge.

Note that the monopole current $k_\mu(x)$ acquires the electric charge and the magnetic monopole is changed to the dyon due to the existence of the θ term in agreement with the Witten effect [9]. *Thus, we completely reproduced the effective dyon action obtained in [6] without any assumptions.*

3. Topological susceptibility and Witten-Veneziano formula

Let us argue that *the dyon configuration is the most relevant one for solving the U(1) problem* in SU(2) QCD by evaluating the topological susceptibility from the dyon configuration appearing in the APEGT with θ-term. To estimate the numerical value of the topological susceptibility, we consider the lattice regularized version of (2.24),

$$S_E = \sum_{x,y} \left(\bar{\beta} + \frac{\theta^2}{\beta} \right) k^\mu(x) D_{\mu\nu}(x-y) k^\nu(y), \quad \bar{\beta} \equiv \frac{1}{2}\left(\frac{4\pi}{g}\right)^2. \tag{3.1}$$

According to the analysis of the monopole action by the inverse Monte-Carlo simulation, the self-mass term of the monopole current is dominant in the low-energy region, e.g., $G_2/G_1 \simeq 0.33$ at the scale 1.7fm where G_1 and G_2 are respectively the self-coupling and the nearest-neighbor coupling of the monopole current [10]. Furthermore, the monopole configuration subject to $|k_\mu(x)| = 1$ is dominant in the low-energy region [11] and the energy density e_θ is written as

$$e_\theta = S_E/V \simeq \left(\bar{\beta} + \frac{\theta^2}{\beta} \right) D(0). \tag{3.2}$$

Therefore, the topological susceptibility χ_E is calculated:

$$\chi_E \equiv \left(\frac{d^2 e_\theta}{d\theta^2} \right)_{\theta=0} \simeq \frac{2}{\beta} D(0). \tag{3.3}$$

The result of quantum perfect lattice action for monopole obtained by Chernodub et al.[10] show $\bar{\beta}D(0) \equiv G_1 = 0.059$ and $\bar{\beta} = 2.49$ at the physical scale $b = 3.8\sigma_{phys}^{-1/2}$. (Note that $b = 1\sigma_{phys}^{-1/2}$ corresponds to 1.7fm, provided that the string tension $\sigma_{phys} \cong (440\text{MeV})^2$ in SU(2) QCD.) By substituting these values into (3.3), the topological susceptibility is determined as

$$\chi_E^{1/4}/\sigma_{phys}^{1/2} = 0.371, \tag{3.4}$$

in units of the string tension σ_{phys}. Remarkably, this estimate reproduces 76% of the full result

$$\chi^{1/4}/\sigma_{phys}^{1/2} = 0.486 \pm 0.010, \tag{3.5}$$

obtained by Teper [12] in the simulation of SU(2)QCD. Note that our result is also consistent with the mass formula for η', the so-called the Witten-Veneziano formula[13].

Thus we conclude that *the dyon, i.e., magnetic monopole with the electric charge proportional to the vacuum angle θ, gives dominant contribution to the topological susceptibility*.

4. Conclusion

In this paper, we have given a short review of how to derive the effective dyon action directly from SU(2) Yang-Mills theory with θ-term by using BRST formalism. By estimating the classical part of the dyon action, we have calculated the topological susceptibility. The obtained value agrees with the numerical result obtained by the recent lattice gauge theory. Thus we have shown that the dyon-like configuration gives a dominant contribution to the topological susceptibility and resolves the U(1) problem.

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