Gauge invariance of the Abelian dual Meissner effect in pure SU(2) QCD

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The dual Meissner effect is described and numerically observed in a gauge-invariant way in lattice Monte-Carlo simulations in pure $SU(2)$ QCD. The squeezing of the non-Abelian electric field between a pair of static quark and anti-quark occurs due to the solenoidal current coming from the gauge-invariant monopole-like quantity. Preliminary results are obtained with respect to the vacuum type of the confinement phase. The $SU(2)$ QCD vacuum seems near the border between the type 1 and the type 2 dual superconductors. The theoretical background of this idea is published in another report [1]. Here we show numerical results in this note.

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1. Introduction

In a previous report [1], we have defined a gauge-invariant monopole-like quantity which we call as 'monopole' through a gauge-invariant Abelian-like field strength. The definition of the Abelian-like field strength $f_{\mu\nu}$ and 'monopole' k_μ on the lattice are the followings:

$$f_{\mu\nu}(s) = n^a_{\mu\nu}(s)F^a_{\mu\nu}(s), \tag{1.1}$$

$$k_\mu(s) = \frac{1}{8\pi}\varepsilon_{\mu\nu\alpha\beta}\Delta_\nu f_{\alpha\beta}(s+\hat{\mu}). \tag{1.2}$$

The purpose of this paper is to show the numerical evidence that the dual Meissner effect occurs in a gauge-invariant way due to the gauge-invariant Abelian-like field strength and 'monopoles'. We do not need any Abelian projection nor any gauge-fixing.

2. Numerical results

2.1 Method

We use RG improved gluonic action [2, 3],

$$S = \beta \left\{ c_0 \sum \mathrm{Tr}\,(plaquette) + c_1 \sum \mathrm{Tr}\,(rectangular) \right\}. \tag{2.1}$$

The mixing parameters are fixed as $c_0 + 8c_1 = 1$ and $c_1 = -0.331$. We adopt the coupling constant $\beta = 1.20$ which corresponds to the lattice distance $a(\beta = 1.20) = 0.0792(2)$ [fm]. The lattice size is 32^4. After 5000 thermalizations, we have taken 2000 thermalized configurations per 100 sweeps for measurements. To get a good signal-to-noise ratio, the APE smearing technique [4] is used for evaluating Wilson loops.
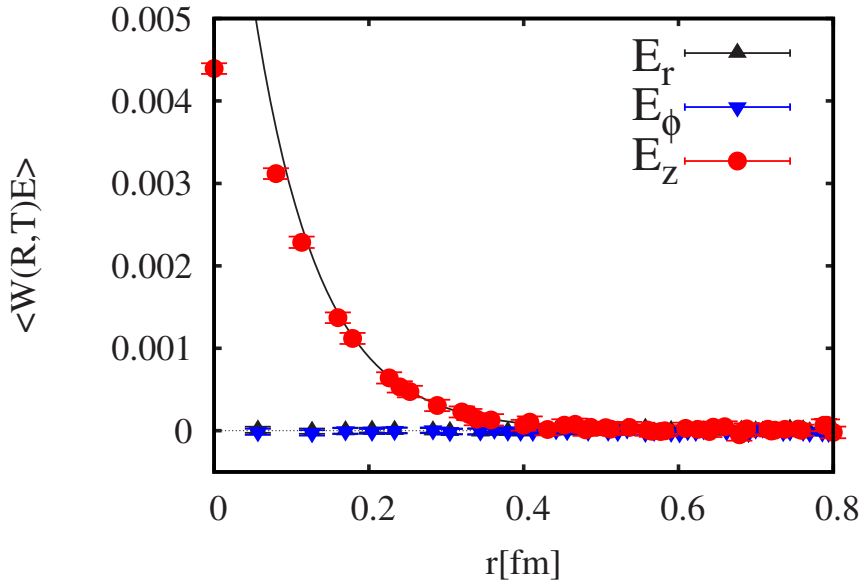

Figure 1: $\vec{E}$ electric field profiles. r is a distance perpendicular to the $Q\bar{Q}$ axis and $W(R \times T = 5 \times 5)$ is used.
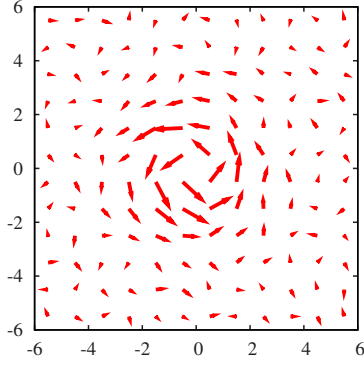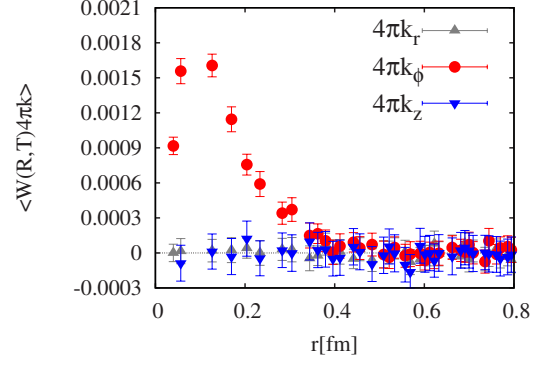
Figure 2: 'Monopole' current distributions.

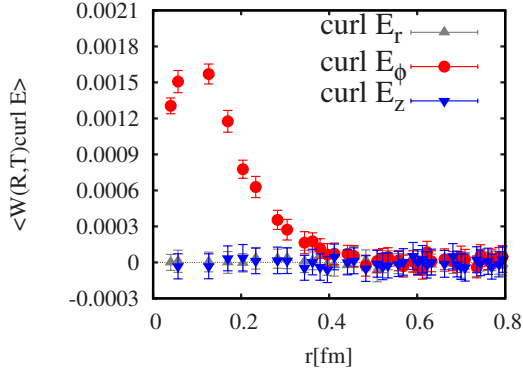Figure 3: Components of 'monopoles' around a static quark pair.

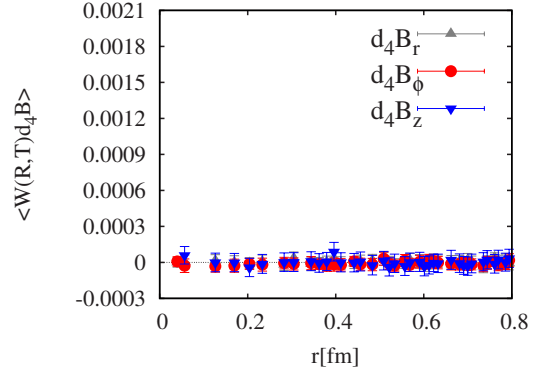Figure 4: Components of curl$\vec{E}$ around a static quark pair.

Figure 5: Components of the magnetic displacement current around a static quark pair.

2.2 Correlations with Wilson loops

Let us try to measure, without any gauge fixing, electric and magnetic flux distributions by evaluating correlations of Wilson loops and the Abelian-like field strengths located in the perpendicular direction to the Wilson-loop plane. First we show in Fig.1 electric field profiles around a quark pair. Only the *z*-component of the electric field is non-vanishing and squeezed. The profiles are studied mainly on a perpendicular plane at the midpoint between the two quarks. Note that electric fields perpendicular to the $Q\bar{Q}$ axis are found to be negligible. The solid line denotes the best exponential fit.

Now let us study the violation of the Bianchi identity with respect to the Abelian-like field strength.

$$\vec{\nabla} \times \vec{E} + \partial_4 \vec{B} = 4\pi \vec{k}. \tag{2.2}$$

The Coulombic electric field coming from the static source is written in the lowest perturbation theory in terms of the gradient of a scalar potential. Hence it does not contribute to the curl of the electric field nor to the magnetic field in the above Bianchi identity Eq.(2.2). The dual Meissner effect says that the squeezing of the electric flux occurs due to cancellation of the Coulombic
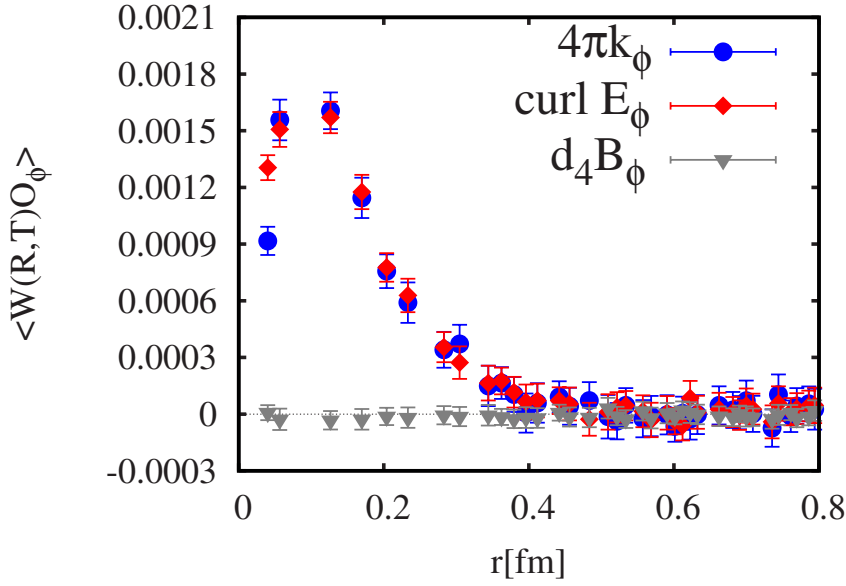
Figure 6: The azimuthal component of 'monopoles' and the magnetic displacement current around a static quark pair.

electric fields and those from solenoidal magnetic currents. It is very interesting to see from Fig.2 that in this gauge-invariant case, the gauge-invariant 'monopole' Eq.(1.2) plays the role of the solenoidal current. This is qualitatively similar to the monopole behaviors in the MA gauge [5, 6, 7, 8].

Let us see also the r dependence of the 'monopole' distribution shown in Fig.3. All r and z components of each term in Eq.(2.2) are almost vanishing consistently with Fig.2. The all components of Eq.(2.2) is plotted in Fig.4 and Fig.5. The magnetic displacement current $\partial_4\vec{B}$ are found to be negligible numerically as similarly as in the MA gauge [5, 6, 9]. We show in Fig.6 and Fig.7 azimuthal components of all three terms of Eq.(2.2) in this gauge-invariant case and in the MA gauge
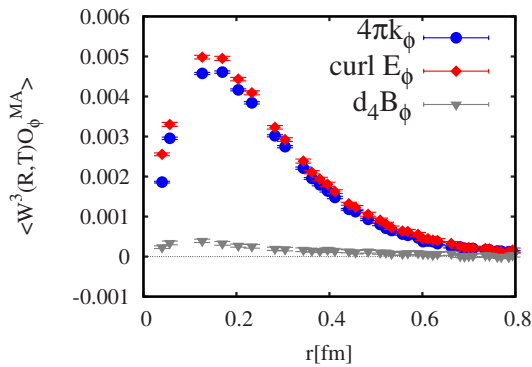

Figure 7: The azimuthal component of the Abelian monopoles and the magnetic displacement current around a static quark pair in the MA gauge.
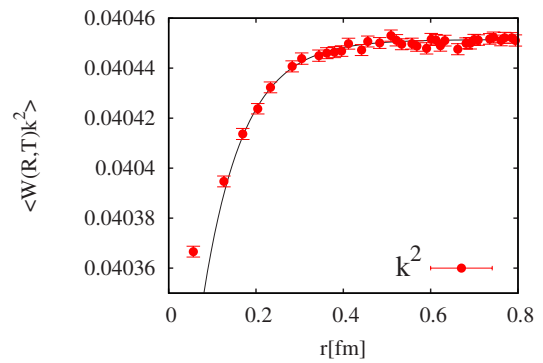

Figure 8: The correlation between the Wilson loop and the squared monopole density. $W(R = 5, T = 5)$ is used.

case[1]. It is interesting that the peak positions of gauge-invariant k_ϕ and k_ϕ^{MA} in the MA gauge look similar around 0.15 [fm], although the height and the shapes seem different.

2.3 The type of the vacuum

Let us next try to fix the type of the vacuum of pure $SU(2)$ QCD. The penetration length λ is determined by making an exponential fit to the electric field flux for large r regions. The best fitting curve is also plotted in Fig. 1 from which we fix the penetration length λ [2]. Next we derive the coherence length ξ. In Ref. [10], we have shown that the coherence length can be fixed by a measurement of the squared monopole density around the $Q\bar{Q}$ pair. The same situation is expected in this gauge-invariant case. The correlation between the Wilson loop and the squared 'monopole' density is plotted in Fig. 8. From the exponential fit, we may fix the coherence length. We get (in Table 1)

$$\lambda = 0.085(5)[fm], \tag{2.3}$$

$$\xi/\sqrt{2} = 0.10(1)[fm]. \tag{2.4}$$

Although the R and T dependences of both lengths are not studied yet, the value of the coherence length looks almost the same as that of the penetration length within the error bars. Hence if the same situations will continue for larger R in the confining string region, the type of the vacuum is fixed to be near the border between the type 1 and the type 2. This is consistent with the result of our previous paper [10] and the result [11] obtained in the MA gauge. It should be stressed that the present result is obtained in a gauge-invariant framework.

Quantity	Fig.	c_0	c_1 [fm]	c_2	$\chi^2/d.o.f.$
$\langle WE_z \rangle$	1	0.009(2)	0.085(5)	0	4.2
$\langle Wk^2 \rangle$	8	0.010(2)	0.10(1)	1.59534(1)	4.1

Table 1: The best fit parameters corresponding to the fits of various quantities by a function $f(r) = c_0 \exp(-r/c1) + c_2$.

3. Conclusions

Monte-Carlo simulations of quenched $SU(2)$ QCD are performed. It has been found that the squeezing of the non-Abelian electric field $\sqrt{(E_i^a)^2}$ occurs and the solenoidal current from the gauge-invariant 'monopoles' is responsible for the flux squeezing. The magnetic displacement current observed previously in Landau gauge [12] has been found to be negligible. Preliminary results have been obtained with respect to the vacuum type of the confinement phase. The $SU(2)$ QCD vacuum seems near the border between the type 1 and the type 2 dual superconductors. This is consistent with the result of our previous paper [10] and the result [11] obtained in the MA gauge.

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[1]In the MA case, we use only the third component W^3 of the non-Abelian Wilson loop as a source.

[2]We have fixed both lengths using a simple exponential function expected in the long-range regions. For details, see Ref. [10].

Present numerical results are not perfect, since the continuum limit, the infinite-volume limit and the real $SU(3)$ case are not studied yet. Nevertheless the results obtained here are very interesting, since they show for the first time the flux squeezing of non-Abelian electric fields is working in a gauge-invariant way due to the dual Meissner effect without performing any Abelian projection.

Acknowledgments

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