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# Eigenmodes of covariant Laplacian in SU(2) Yang–Mills vacuum: higher representations†

**J. Greensite**
*The Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark;*
*Physics and Astronomy Dept., San Francisco State University, San Francisco, CA 94117, USA*
*E-mail:* greensit@stars.sfsu.edu

**Š. Olejník**
*Institute of Physics, Slovak Academy of Sciences, SK–845 11 Bratislava, Slovakia*
*E-mail:* stefan.olejnik@savba.sk

**M. I. Polikarpov, S. N. Syritsyn***
*ITEP, B. Cheremushkinskaya 25, Moscow 117259, Russia*
*E-mail:* polykarp@itep.ru, syritsyn@itep.ru

**V. I. Zakharov**
*Max-Planck Institüt für Physik, Föhringer Ring 6, D-80805 Munich, Germany*
*E-mail:* xxz@mppmu.mpg.de

The study of lowest eigenmodes of the covariant Laplacian in fundamental representation of the gauge group revealed their specific localization properties. These may bear information on confinement of fundamental scalar particles in SU(2) Yang–Mills vacuum. It was expected that scalar particle eigenmodes in other representations would be localized in different physical volumes. However simulations show strikingly different results for the adjoint and higher ($J = 3/2$) representations. Apart from much higher extent of localization, we find evidence of different scaling behavior of localized eigenmodes.

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*Speaker.

## 1. Introduction

A preceding talk at this meeting [1] presented our results [2] on localization properties of the covariant Laplacian in the fundamental representation. The operator studied is the simplest discretization of the covariant Laplacian:

$$(\Delta\phi)_x^\alpha = \sum_\mu \left[ U_{x,\mu}^{\alpha\beta} \phi_{x+\hat{\mu}}^\beta - 2\phi_x^\alpha + U_{x,-\mu}^{\alpha\beta} \phi_{x-\hat{\mu}}^\beta \right], \tag{1.1}$$

where $U_{x,\mu}^{\alpha\beta}$ is covariant transporter in the given representation.

To investigate localization we calculate the Inverse Participation Ratio of the probability density of a wave function:

$$\mathrm{IPR} = \frac{V\sum_x \rho^2(x)}{\left(\sum_x \rho(x)\right)^2}, \quad \rho(x) = \phi^\dagger(x)\phi(x) \tag{1.2}$$

which allows us to estimate the "mean" localization volume at given parameters and reveal its scaling properties.

## 2. Adjoint representation

The adjoint covariant transporter is (with $U_{x,\mu}$ being in the fundamental representation)

$$\left[U_{x,\mu}^{\alpha\beta}\right]_{adj} = \frac{1}{2}\mathrm{Tr}\left[U_{x,\mu}\sigma^\alpha U_{x,\mu}^\dagger \sigma^\beta\right] \tag{2.1}$$

which is $SO(3)$ group-valued and has trivial image of the center subgroup. IPR values for the lowest eigenmodes (e.m.'s) are shown in Fig. 1 which covers a wide range in weak couplings ($\beta = 2.10 \ \ldots \ 2.60$) and lattice volumes. The most striking fact is the scaling of IPR with lattice spacing $a$:

$$a^2 \cdot \mathrm{IPR} \sim V_{tot}, \qquad V_{loc} \approx const \cdot a^2. \tag{2.2}$$

The shape of the localization region turns out to be approximately spherical, as clearly seen from density visualizations [3]. The radius of support of any localized mode shrinks to zero as $a \to 0$.

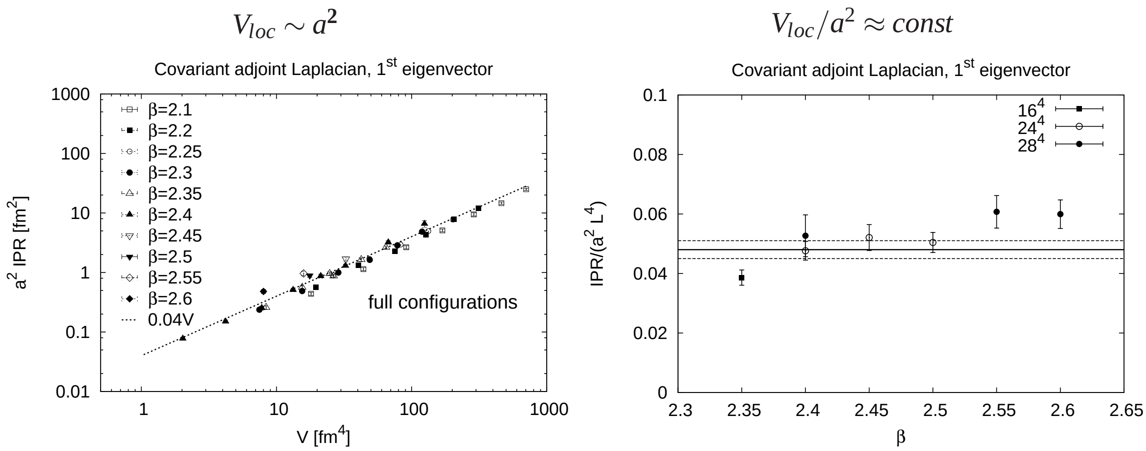



**Figure 1:** Adjoint Laplacian eigenmodes, zero temperature.

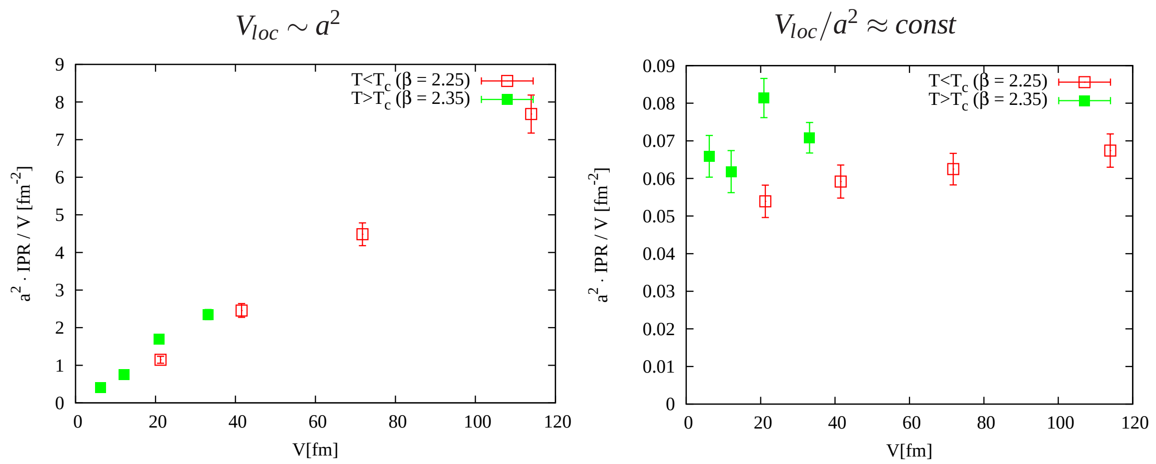



**Figure 2:** Adjoint Laplacian eigenmodes, below and above the phase transition.

The same analysis is performed for finite temperature field configurations. We take lattices with time extension of $L_t = 4$ (the critical point is at $\beta_c \approx 2.30$), while the space extension of our lattices varies between $L_s = 16 \ \ldots \ 28$ lattice spacings. To see the effect of crossing the phase transition we used values of $\beta = 2.25$ and $\beta = 2.35$. For any point 20 independent configurations are sampled, sufficient to reveal the qualitative behavior of IPRs. Figures 1 and 2 show the same scaling behavior of IPR, hence the same scaling of localization volume remains valid both below and above the deconfinement temperature $T_c$.

Despite the similarity of results in confinement and deconfinement phases the localization is related to infrared phenomena. Dimensional analysis of localization volume implies that

$$V_{loc} = const \cdot a^2 = \frac{a^2}{\Lambda^2_{QCD}} \tag{2.3}$$

and $V_{loc}$ is determined by some mixed scale.

Now the following question is addressed: Could such localization result from ordinary gaussian fluctuations, or is it due to confining features of the quantum vacuum? To check this, we simulate the model of gauge fields coupled to Higgs fields in the fundamental representation. It is known to have two phases: confinement-like and Higgs-like [4, 5], but any two points in the phase diagram can be joined by a line along which the free energy is entirely analytic. The transition between the two phases is the vortex depercolation transition: In the confinement-like phase vortices are abundant and percolate over the whole lattice volume, while in the Higgs-like phase the vortex density is small and vortices do not percolate [6 – 8]. The model action is given by

$$S = \beta \sum_{plaq} \frac{1}{2} \mathrm{Tr}\left[UUU^\dagger U^\dagger\right] + \gamma \sum_{links} \frac{1}{2} \mathrm{Tr}\left[\phi^\dagger(x) U_\mu(x) \phi(x+\hat{\mu})\right]. \tag{2.4}$$

At $\beta = 2.1$ the phase transition occurs at $\gamma = 0.9$. Two values of $\gamma$ are taken for comparison: $\gamma = 0.7$ (confinement-like) and $\gamma = 1.2$ (Higgs-like). Fig. 3 shows a drastic reduction of IPR in the Higgs phase. Other tests [2] also show that in the Higgs-like phase the lowest eigenmodes are much more extended; a crucial point is that the falloff of the density outside the support is not exponential in the Higgs-like phase, and this is inconsistent with Anderson localization.

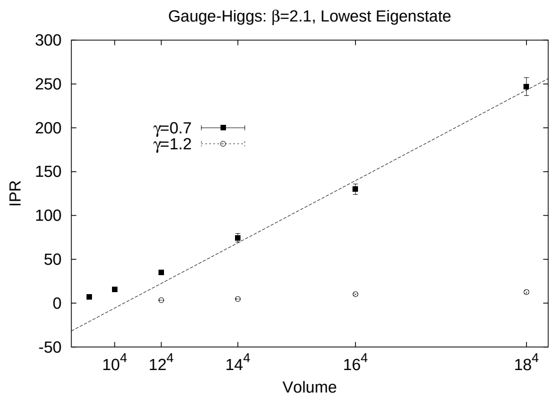



**Figure 3:** Adjoint Laplacian eigenmodes in the gauge-Higgs model, both confinement-like and Higgs-like phases.

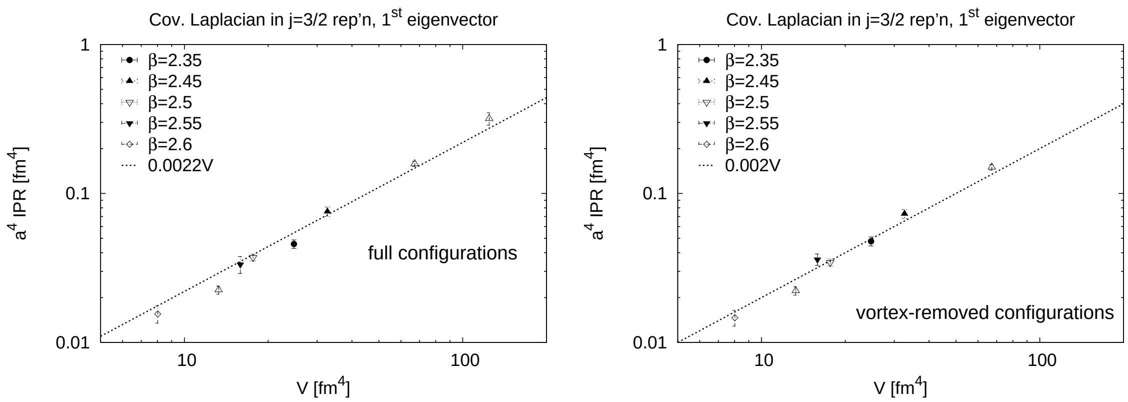
## 3. $J = 3/2$ representation

Another representation studied is a complex 4-dimensional, or isospin $J = 3/2$ color SU(2) representation. The center subgroup is non-trivial and the effect of P-vortices could be separated when one compares the original gauge field and the one modified via the de Forcrand–D'Elia trick [9]. The scaling behavior of IPR is shown in Fig. 4a. The localization volume diminishes with lattice spacing even more quickly than for the adjoint representation. From Fig. 4 one concludes

$$a^4 \cdot \mathrm{IPR} \sim V_{tot}\,, \qquad V_{loc} \approx const \cdot a^4. \tag{3.1}$$

Density plots in [3] show that the support of these localized modes is again spherical. All geometrical parameters of localization seem to be governed only by the ultraviolet scale $\Lambda_{QCD}$.

Unlike the case of fundamental representation, the eigenmodes of $J = 3/2$ Laplacian are localized on modified (vortex-removed) fields almost as sharply as on original fields (see Fig. 4b).




(a) (b)

**Figure 4:** Laplacian eigenmodes for $J = 3/2$ representation: original gauge (a) and modified (b) fields, zero temperature.

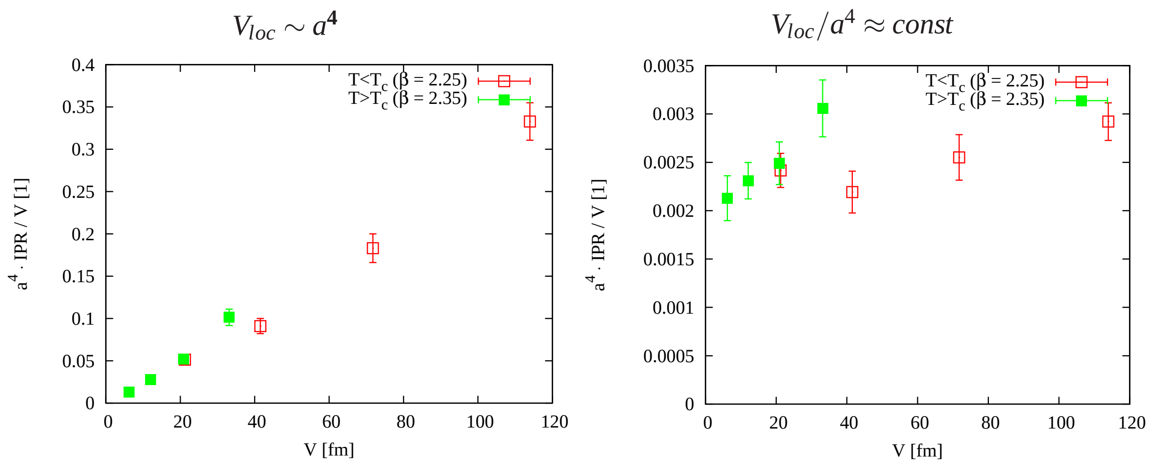



**Figure 5:** Laplacian eigenmodes for $J = 3/2$ representation, finite temperature.

This is an evidence in favor of some other reason for localization than in case of the adjoint and fundamental covariant Laplacians.

The same finite-temperature trajectories as for the adjoint representation were used to study the $J = 3/2$ Laplacian eigenmodes. The scaling of IPR looks very similar for confinement and deconfinement phases; it shows that it is ultraviolet fluctuations that are responsible for localization in this case.

## 4. Summary

Naively one would expect similar localization behavior of eigenmodes of covariant Laplacians in different representations of the gauge group (up to differences in sizes of localization volumes due to different interaction strengths), but the naive expectation is not fulfilled. The presented data demonstrate that at least two covariant Laplacians in representations other than the fundamental posess dramatically different features.

Our results are the following:

- The adjoint covariant Laplacian eigenmodes are localized in volumes which shrink as $V_{loc} \sim a^2$ in the continuum limit. Both scales, infrared and ultraviolet, seem to govern the localization.

- Localization in the adjoint representation, as in the fundamental representation, is related to the presence of center vortices. Vortex removal by the procedure of de Forcrand–D'Elia does not work here, but in the gauge-Higgs model, in which the vortex content is regulated by the gauge-Higgs coupling constant, strong localization is observed in the vortex-abundant (confinement-like) phase, while it is absent in the Higgs-like phase, where the vortex density is low.

- The isospin $J = 3/2$ representation e.m.'s are localized in volumes $V_{loc} \sim a^4$. Vortex removal does not affect this phenomenon.

- The deconfinement transition doesn't influence the character of localization in the adjoint and $J = 3/2$ representations.

The relation of localization of covariant-Laplacian eigenmodes to confinement is questionable in the light of our results, in particular of the dependence of the degree of localization on the group representation of the Laplacian. It may be that localized eigenmodes in different group representations are probing different features and different length scales of the QCD vacuum.

## References

[1] J. Greensite, *Localized eigenmodes of the covariant lattice Laplacian*, in proceedings of *Lattice 2005*, PoS(LAT2005)325.

[2] J. Greensite, Š. Olejník, M. Polikarpov, S. Syritsyn, and V. Zakharov, *Localized eigenmodes of covariant Laplacians in the Yang–Mills vacuum*, *Phys. Rev. D* **71** (2005) 114507 [hep-lat/0504008].

[3] http://lattice.itep.ru/localization; http://dcps.sav.sk/localization.

[4] E. H. Fradkin and S. H. Shenker, *Phase diagrams of lattice gauge theories with Higgs fields*, *Phys. Rev.* D **19** (1979) 3682; K. Osterwalder and E. Seiler, *Gauge field theories on the lattice*, *Annals Phys.* **110**, 440 (1978).

[5] J. Greensite, Š. Olejník, and D. Zwanziger, *Coulomb energy, remnant symmetry, and the phases of non-Abelian gauge theories*, *Phys. Rev. D* **69** (2004) 074506 [hep-lat/0401003].

[6] M. N. Chernodub, F. V. Gubarev, E.-M. Ilgenfritz, and A. Schiller, *Embedded topological defects in hot electroweak theory: A lattice study*, *Phys. Lett.* **B434** (1998) 83 [hep-lat/9805016].

[7] K. Langfeld, *Confinement versus color superconductivity: A lattice investigation*, [hep-lat/0109033]; *Lattice effective theory and the phase transition at finite densities*, [hep-lat/0212032].

[8] R. Bertle, M. Faber, J. Greensite, and Š. Olejník, *Center dominance in SU(2) gauge-Higgs theory*, *Phys. Rev.* D **69** (2004) 014007 [hep-lat/0310057].

[9] Ph. de Forcrand and M. D'Elia, *Relevance of center vortices to QCD*, *Phys. Rev. Lett.* **82** (1999) 4582 [hep-lat/9901020].