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Casimir Scaling of domain wall tensions in the deconfined phase of D=3+1 SU(N) gauge theories.

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We perform lattice calculations of the spatial 't Hooft k-string tensions, $\tilde{\sigma}_k$, in the deconfined phase of SU(N) gauge theories for $N = 2,3,4,6$. These equal (up to a factor of T) the surface tensions of the domain walls between the corresponding (Euclidean) deconfined phases. For $T \gg T_c$ our results match on to the known perturbative result, which exhibits Casimir Scaling, $\tilde{\sigma}_k \propto k(N-k)$. At lower T the coupling becomes stronger and, not surprisingly, our calculations show large deviations from the perturbative T-dependence. Despite this we find that the behaviour $\partial\tilde{\sigma}_k/\partial T \propto k(N-k)$ persists very accurately down to temperatures very close to T_c. Thus the Casimir Scaling of the 't Hooft tension appears to be a 'universal' feature that is more general than its origin in the low order high-T perturbative calculation. We observe the 'wetting' of these k-walls at $T \simeq T_c$. Our calculations show that as $T \to T_c$ the magnitude of $\tilde{\sigma}_k(T)$ decreases rapidly.

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1. Introduction

In the Euclidean formulation of SU(N) gauge theories at finite T, deconfinement is associated with the spontaneous breaking of a Z_N centre symmetry. The obvious order parameter is the trace of the Polyakov loop, l_p. In the deconfined phase the effective potential for the Polyakov loop trace averaged over the volume, $\bar{l}_p$, has its minimum at $\bar{l}_p \neq 0$, so the symmetry is spontaneously broken and there are N possible deconfined phases. If $\bar{l}_p \propto z_k = e^{2\pi i \frac{k}{N}}$ we label the phase by k. When two of these phases, k_1 and k_2 say, co-exist they will be separated by a domain wall whose surface tension σ_W^k will depend on $k = k_1 - k_2$, as well as on N and T. The tension of these domain walls is equal, up to a factor of T, to the spatial 't Hooft string tension, which is in principle related to confinement.

At high T one can calculate σ_W^k in perturbation theory. To two loops one finds [2, 3]:

$$\sigma_W^k = k(N-k)\frac{4\pi^2}{3\sqrt{3}}\frac{T^3}{\sqrt{g^2(T)N}}\{1-\tilde{c}_2 g^2(T)N\} \tag{1.1}$$

where $\tilde{c}_2 \simeq 0.09$.

The factor $k(N-k)$ is the k dependence of the Casimir, $Tr_{\mathcal{R}}T^aT^a$, where $\mathcal{R}$ is the totally antisymmetric representation of a product of k fundamentals of SU(N). It is the factor one obtains when calculating the Coulomb interaction between sources in such a representation. In D=1+1 SU(N) gauge theories, the tension of the confining k-string that connects such sources has precisely this dependence. There are old speculations [5] that this 'Casimir Scaling' holds in D=3+1 and numerical calculations show that it is a good (but not exact) approximation [6]. A question we will try to answer in this paper is whether the Casimir Scaling in eqn(1.1) survives at very much lower T. This question connects to the role of Casimir Scaling in confinement, since these domain walls are closely related to the centre vortices that provide a possible mechanism for confinement[8, 9].

2. Preliminaries

We discretise Euclidean space-time to a periodic hypercubic lattice. The $\mu = 0$ direction is the temperature direction and the domain wall spans the $L_1 \times L_2$ torus.

Our order parameter will be based on the Polyakov loop l_p. Above T_c there are N degenerate phases in which $\langle l_p \rangle = z_k c(\beta)$ where $c(\beta)$ is a real-valued renormalisation factor. They can co-exist at any $T \geq T_c$ and will be separated by domain walls. If l_p in the two phases differs by a factor z_k we refer to the domain wall as a k-wall.

To study a k-wall we use a 'twisted' plaquette action to enforce the presence of a single domain wall. The twisted action is defined by

$$S_k = \sum_p \left(1 - \frac{1}{N}\mathrm{ReTr}\{z(p)U_p\}\right), \tag{2.1}$$

where $z(p) = 1$ for all plaquettes except

$$z(p = \{\mu\nu, x_\mu\}) = z_k = e^{2\pi i \frac{k}{N}} \qquad \mu\nu = 03;\ x_0 = x_0{}',\ x_3 = x_3{}',\ x_{1,2} = 1, ..., L_{1,2}. \tag{2.2}$$

That is to say, the plaquettes in the entire (0,3)-plane at $x_0 = x_0{}'$ and $x_3 = x_3{}'$ are multiplied by $z_k \in Z_N$. The Polyakov loops on either side of the plane will differ by a factor of z_k. Periodicity in x_3 then demands that at some x_3 the Polyakov loop must suffer a compensating factor of $z_k{}^\dagger$ – a domain wall. Thus we ensure that each configuration possesses at least one k-wall.

We calculate the average action with and without a k-twist as defined above. The difference is

$$\Delta S_k \equiv \langle S_k \rangle - \langle S_0 \rangle = \frac{\partial \ln Z_0}{\partial \beta} - \frac{\partial \ln Z_k}{\partial \beta} = \frac{\partial}{\partial \beta} \frac{F_k - F_0}{T} = \frac{\partial}{\partial \beta} \frac{\sigma_W^k A}{T} \tag{2.3}$$

where σ_W^k is the surface tension of the domain wall and $A = a^2 L_1 L_2$ is its area.

We see from eqn(1.1) that we expect

$$\Delta S_k = \frac{\partial}{\partial \beta} \frac{\sigma_W^k A}{T} \stackrel{T \to \infty}{=} \alpha(L_0) \frac{k(N-k)}{\sqrt{N}} \frac{4\pi^2}{3\sqrt{3}} \frac{L_1 L_2}{L_0^2} \frac{\partial}{\partial \beta} \frac{1}{g(T)} \tag{2.4}$$

when leading order perturbation theory is accurate. The factor of $\alpha(L_0)$ contains the $O(a^2T^2) = O(1/L_0^2)$ lattice correction, obtained by a numerical evaluation of the expression given in [4].

3. Results

3.1 Surface tension

Our most extensive results are for SU(4) with $L_0 = 4$. We show the values of $\Delta S_W^{k=1}$ for these calculations in Fig. 1, compared to the two-loop perturbative expectations using a mean-field improved coupling and the bare coupling. We also include a ‘good’ coupling evaluated at low energy scales: the Schrodinger functional coupling which has been calculated non-perturbatively in [11]. We plot the ratios $\Delta S_w^2/\Delta S_w^1$ in Fig. 3.

We obtained results at one additional point, not included in the graphs, at $\beta = 20$, corresponding to $T \sim 1000 T_c$. Here we found excellent agreement with perturbation theory, as expected.

In Table 1 we list our results for ΔS_W^k for SU(4) with $L_0 = 5$ and for SU(2), SU(3), and SU(6). We include our results for SU(4) with $L_0 = 4$ at the same temperatures for comparison.

For SU(4) with $L_0 = 4$, as $T \to T_c$ we find ΔS_W^k grows much more rapidly than the perturbative prediction, reaching a factor of 18 at $T \simeq 1.02 T_c$. This tells us that $\partial \sigma_W^k/\partial T$ becomes much larger than the low-order perturbative expectation as $T \to T_c$, implying that $\sigma_W^k(T)$ is increasingly suppressed relative to its perturbative value as we approach T_c. The ratio of the ΔS_W^k satisfies Casimir Scaling to good accuracy, though there is some evidence for a discrepancy below $1.5 T_c$.

To investigate the continuum limit we compare to $L_0 = 5$. The discrepancy with perturbation theory is the same at $T \simeq 1.88 T_c$ and similar at $T \simeq 1.02 T_c$. It is clear that a large and growing mismatch with perturbation theory as $T \to T_c$ is a feature of the continuum theory. The ratio of the ΔS_W^k continues to be close to Casimir Scaling, so that is also a property of the continuum theory.

For SU(6) we observe at both values of T precisely the same discrepancy with perturbation theory as we saw for SU(4) at the same value of L_0. In addition the ΔS_W^k ratios continue to satisfy Casimir Scaling. Taken together this tells us that the derivative of the domain wall tension has no factors of k and N except for the Casimir scaling factor $k(N-k)$ and its dependence on the ’t Hooft coupling, $g^2 N$. Our SU(2) and SU(3) calculations show a very similar discrepancy with perturbation theory. Thus the suppression of $\sigma_W^k(T)$ as $T \to T_c$ is largely independent of N.

To estimate the suppression of σ_W^k we can in principle calculate it by interpolating $\partial\sigma_W^k/\partial\beta$ in β and then integrating from large β down to the desired value of T, but our calculations are not dense enough in β for this. We can obtain a qualitative picture by assuming some functional form for $\partial\sigma_W^k/\partial\beta$ and fitting it to the calculated values. Choosing the $L_0 = 4$ SU(4) calculation, we make a fit using a simple modification of the one-loop formula for the k-wall free energy:

$$\frac{F_W^k}{L_1 L_2 T} = [1\ \mathrm{loop}] + a\exp(-b\sqrt{\beta_I - \beta_{Ic}}) \tag{3.1}$$

Using this fit we obtain the T-dependence of σ_W^k shown in Fig. 2. We display the uncertainty from the errors in the fitted parameters, but it is clear that we cannot reliably estimate the systematic error inherent in the choice of fitting function. The qualitative picture is that σ_W^k is strongly suppressed at $T \leq 1.5T_c$. This suggests that it may reach zero at some $T_{\tilde{H}}$ somewhat below T_c, causing a second-order phase transition due to the condensation of spatial 'tHooft strings.

3.2 Profile

We can average the Polyakov loop over the transverse coordinates to obtain the profile $\bar{l}_p(x_3)$. Comparing with the perturbative prediction [2, 4] we find good agreement down to $T \simeq 1.88T_c$. However, at $T \simeq 1.02T_c$ the profile is very far from the perturbative expectation. This observation makes it all the more remarkable that we continue to see Casimir Scaling at such very low T.

3.3 Wetting

A k-wall can interpolate between two deconfined phases by passing through the origin of the complex $l_p(\vec{n})$ plane, i.e. the confined phase. Whether this will happen or not depends on the relevant surface tensions. We investigated this by a series of runs very near T_c in SU(4) and SU(6). We found that all the walls split into a pair of confined-deconfined walls ('wetting') over a range of $\sim 0.01T_c$ in T. This small range occurs because the latent heat is much larger than the domain wall tensions, which suppresses the breaking up unless T is extremely close to T_c.

4. Conclusions

We have shown that surface tensions of domain walls seperating different deconfined phases are close to Casimir Scaling, i.e.

$$\sigma_W^k = k(N-k)\, f(g^2(T)N, T/T_c)\, T^3, \tag{4.1}$$

to a good approximation. The surface tensions are strongly suppressed as $T \to T_c$. The domain wall profile is far from the perturbative prediction near T_c. We also observed 'wetting' of the domain walls very near to T_c.

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N	aT	k	Prediction	$\Delta S^k_w / L_x L_y$	$\Delta S^k_w / \Delta S^1_w$	CS	T/T_c
2	0.25	1	0.1042	0.3089(84)			1.88
3	0.25	1	0.1032	1.380(40)			1.02
3	0.25	1	0.0936	0.2716(67)			1.88
4	0.25	1	0.0867	1.615(30)			1.02
		2	0.1157	2.141(30)	1.326(29)	1.333	1.02
4	0.25	1	0.0791	0.2200(56)			1.88
		2	0.1055	0.2957(57)	1.344(31)	1.333	1.88
4	0.20	1	0.0514	0.739(27)			1.02
		2	0.0685	0.982(28)	1.329(56)	1.333	1.02
4	0.20	1	0.0467	0.1292(54)			1.88
		2	0.0622	0.1743(60)	1.350(57)	1.333	1.88
6	0.20	1	0.0381	0.488(25)			1.02
		2	0.0610	0.847(27)	1.74(9)	1.60	1.02
		3	0.0687	1.015(29)	2.08(10)	1.80	1.02
6	0.20	1	0.0345	0.098(9)			1.88
		2	0.0552	0.159(10)	1.62(14)	1.60	1.88
		3	0.0621	0.171(10)	1.74(15)	1.80	1.88

Table 1: Results for ΔS^k_w. The prediction is the two-loop perturbative expectation, using a mean-field improved coupling.

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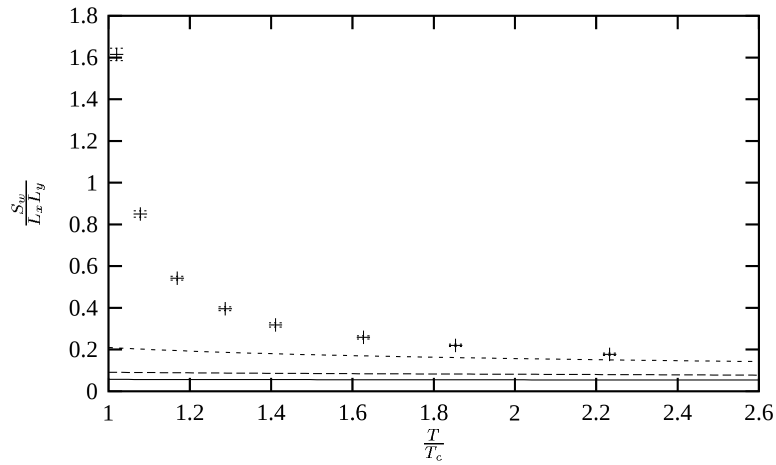

Figure 1: Action per unit area of the $k = 1$ domain wall in SU(4) with $aT = 0.25$. Monte Carlo values, $+$, compared with perturbation theory based on various couplings: $g^2(a)$, solid line, $g_I^2(a)$, long dashed line, $g_{SF}^2(T)$, short dashed line.

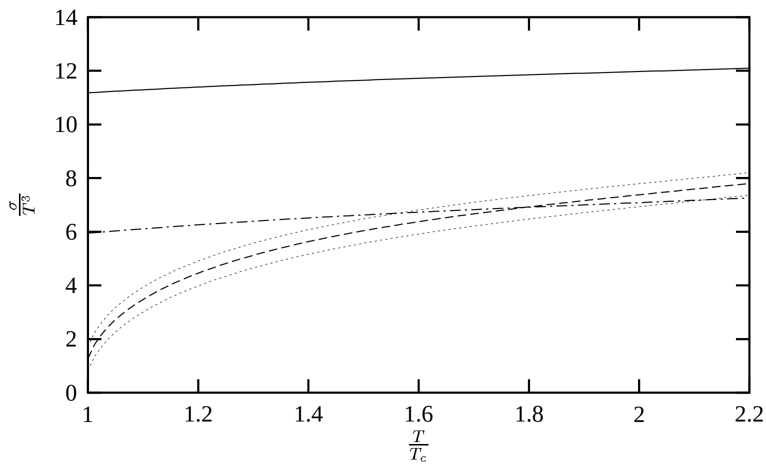

Figure 2: Surface tension in units of T, using the functional form in eqn(3.1) fitted to our results. For comparison we show the 1-loop (solid line) and 2-loop (dot-dashed line) perturbative results using a mean field improved coupling. All for the $k = 1$ wall in SU(4).

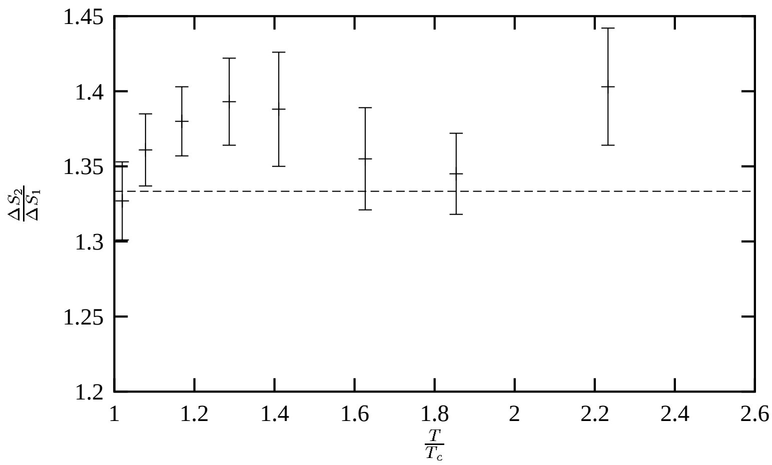

Figure 3: Ratio $\Delta S_w^2 / \Delta S_w^1$ in SU(4).