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Maximal twist and the spectrum of quenched twisted mass lattice QCD

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Results on the hadron masses for a degenerate doublet of up and down quarks from quenched twisted mass lattice QCD at maximal twist are presented. Two definitions of maximal twist are used and the hadron masses for these definitions are compared. Mass splittings within the $\Delta(1232)$ multiplet due to flavor breaking effects are discussed.

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1. Introduction

With twisted mass lattice QCD(tmQCD) [1], it became possible to perform lattice simulations with Wilson type quarks at small quark masses. This is because of the fact that the Wilson Dirac operator with a twisted mass term is free of unphysical zero eigenmodes as long as the quark mass and the twist angle are nonzero. Another extremely useful property of tmQCD is that when the twist angle equals $\frac{\pi}{2}$ (maximal twist), discretization errors of $O(a^{2k+1}); k = 0, 1, 2, \ldots$ of correlation functions of gauge invariant, multiplicatively renormalizable, multi-local operators are either automatically absent or could be removed by taking a simple averaging procedure[2]. This is particularly simple in the case of extracting masses from 2-point correlators, since the improvement will be automatic and no averaging is needed. These features make tmQCD at maximal twist an attractive method to study the hadronic spectrum. An important element in this program is the definition of maximal twist and how the uncertainty in this definition could affect the expected reduction of discretization errors. Initial quenched simulations [3] used the condition of maximal twist from the vanishing of the pion mass as computed in the untwisted action (Wilson maximal twist). Improved results were obtained [3][4] using Wilson maximal twist at not so light quark masses. However, for very light quark masses (close to the interesting physical up and down quark masses), undesired discretization effects became evident[5][6]. In addition tmQCD chiral perturbation theory [7][8] as well as a study of discretization errors in tmQCD á la Symanzik [9] argued for the need for an optimal determination of the maximal twist condition in the light quark mass regime. Improved definitions of maximal twist [11][12] have been used in recent simulations [6][10][11], using PCAC or the vanishing of wrong parity matrix elements(parity maximal twist). For more discussion, see [13].

In this work, results for the quenched hadron spectrum from a degenerate doublet of up and down quarks are presented for various quark masses and at different lattice spacings. Simulations were performed implementing both the Wilson maximal twist and parity maximal twist definitions and results are compared.

2. Action at Maximal Twist

The action is given by

$$S[\psi, \bar{\psi}, U, \beta] = S_F[\psi, \bar{\psi}, U] + S_g[\beta, U], \tag{2.1}$$

where S_g is the standard Wilson plaquette action for the gauge field at coupling β, while S_F is the tmQCD action for a degenerate doublet ψ of the up and down quarks. In the twisted basis

$$S_F[\psi, \bar{\psi}, U] = a^4 \sum_x \bar{\psi}(x) \left(M + i\mu_q \gamma_5 \tau^3 + \not{D}_{\mathrm{W}}\right) \psi(x), \tag{2.2}$$

where

$$\not{D}_{\mathrm{W}} = \frac{1}{2} \sum_\nu \left(\gamma_\nu \nabla_\nu + \gamma_\nu \nabla^*_\nu - a \nabla^*_\nu \nabla_\nu\right),$$

$$\nabla_\nu \psi(x) = U_\nu(x) \psi(x + a\hat{\nu}) - \psi(x), \qquad \nabla^*_\nu \psi(x) = \psi(x) - U^\dagger_\nu(x - a\hat{\nu}) \psi(x - a\hat{\nu}).$$

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β	#sites	#configurations	$aM_{critical}$	$a\mu_q$	$am_q^{physical}$
5.85	$20^3 \times 40$	1000	-0.9071	0.0376	$\sim m_s$
				0.0188	$\sim m_s/2$
				0.012527	$\sim m_s/3$
				0.00627	$\sim m_s/6$
		600	-0.8965	0.0376	$\sim m_s$
			-0.0971	0.0188	$\sim m_s/2$
			-0.9110	0.01252	$\sim m_s/3$
			-0.9150	0.00627	$\sim m_s/6$
6.0	$20^3 \times 48$	1000	-0.8135	0.030	$\sim m_s$
				0.015	$\sim m_s/2$
				0.010	$\sim m_s/3$
				0.005	$\sim m_s/6$
		600	-0.8110	0.030	$\sim m_s$
			-0.8170	0.015	$\sim m_s/2$
			-0.8195	0.010	$\sim m_s/3$
			-0.8210	0.005	$\sim m_s/6$
6.2	$28^3 \times 56$	200	-0.7363	0.02165	$\sim m_s$
				0.01083	$\sim m_s/2$
				0.00722	$\sim m_s/3$
				0.00361	$\sim m_s/6$

Table 1: Parameters of the simulations. The bare quark mass $a\mu_q$ values were chosen such that the physical quark mass $am_q^{physical}$ corresponds to specific fractions of the strange quark mass. For Wilson maximal twist, the value of $aM_{critical}$ is independent of the bare quark mass value.

When the mass parameter $M = M_{critical}$, we are at maximal twist and the bare quark mass equals $a\mu_q$. Simulations are performed with $M_{critical}$ defined by the value that corresponds to a massless pion in the standard Wilson action (Wilson maximal twist)[3], and with $M_{critical}$ defined by the value when the wrong parity mixing between the physical vector and pseudoscalar vanishes (parity maximal twist), i.e.

$$\sum_{\vec{x}} \left\langle V_\nu^-(\vec{x},t) P^+(0,0) \right\rangle = 0. \tag{2.3}$$

3. Simulations

In Table 1, the simulation parameters are listed. The values of $M_{critical}$ for the parity definition of maximal twist were tuned at each μ_q such that the condition in Eq.2.3 is satisfied. Standard local interpolating fields for the charged pion, the charged ρ , $J^P = (\frac{1}{2})^\pm$ baryons and $J^P = (\frac{3}{2})^\pm$ baryons are used to obtain the ground state masses[6]. Using the PCAC relation, the charged pion decay constant can be obtained from

$$f_\pi = \frac{2\mu_q}{m_\pi^2} |\left\langle 0|P^\pm|\pi\right\rangle|. \tag{3.1}$$

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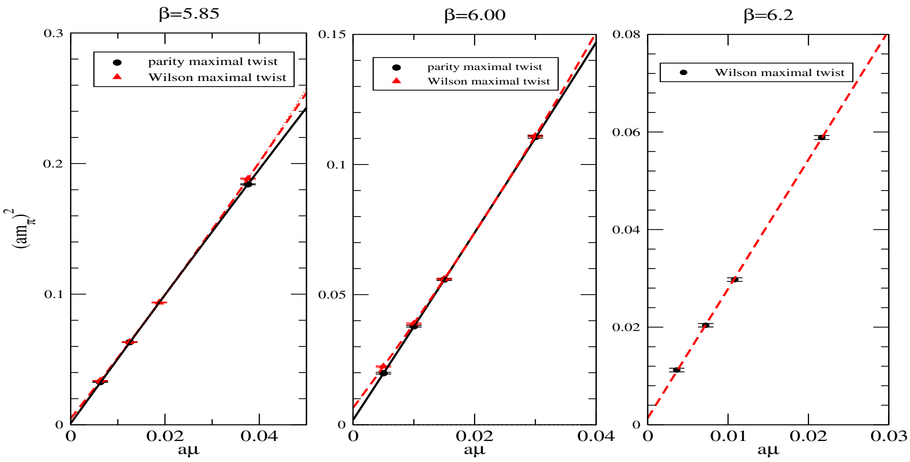

Figure 1: The charged pion mass squared (in lattice units) as a function of the bare quark mass $a\mu_q$.

4. Results

In Figure 1, the charged pion mass squared is plotted as a function of the bare quark mass. The parity definition of maximal twist shows the expected linear dependence at light quark masses with am_π approximately vanishing in the chiral limit. A better scaling behavior of f_π is also observed with the parity definition of maximal twist as shown in Figure 2, while the scaling of the rho mass is equally good in both definitions if one takes into account the large statistical errors. For the rho mass, results extracted using the 4^{th} component of the tensor operator are also shown which were found, in some cases, to have small statistical errors. In Figure 3, results for the Nucleon and Delta masses are shown for both definitions of maximal twist. These are about 10% higher than the physical values, a result that might be expected due to quenching.

The twisted mass action includes flavor breaking effects due to the Wilson term. These could be seen as a mass splitting among isospin multiplets. In case of the Δ multiplet, there are no disconnected diagrams and one can study the mass splitting among the four Δ states. On the average, for a degenerate doublet of the up and down quarks, the up quark propagator $U(x,y)$ and the down quark propagator $D(x,y)$ are not equal but are related by [6]

$$U(x,y) = \gamma_5 D(y,x)^\dagger \gamma_5. \tag{4.1}$$

Using this relation, one expects the following mass relation among the Δ states,

$$(M_{\Delta^{++}} = M_{\Delta^-} = M_{\Delta^{++,-}}) \neq (M_{\Delta^+} = M_{\Delta^0} = M_{\Delta^{+,0}}). \tag{4.2}$$

In Figure 4, the measured mass splittings are shown for $\beta = 6.0$. From these results, one concludes that there could be a hint of flavor breaking, however, these are hidden by large statistical errors. The flavor breaking effects between the charged and neutral pion need the computation of disconnected diagrams which has been calculated in [14] [15]. A recent study of tmQCD with a strange quark also includes some discussion of flavor breaking[16].

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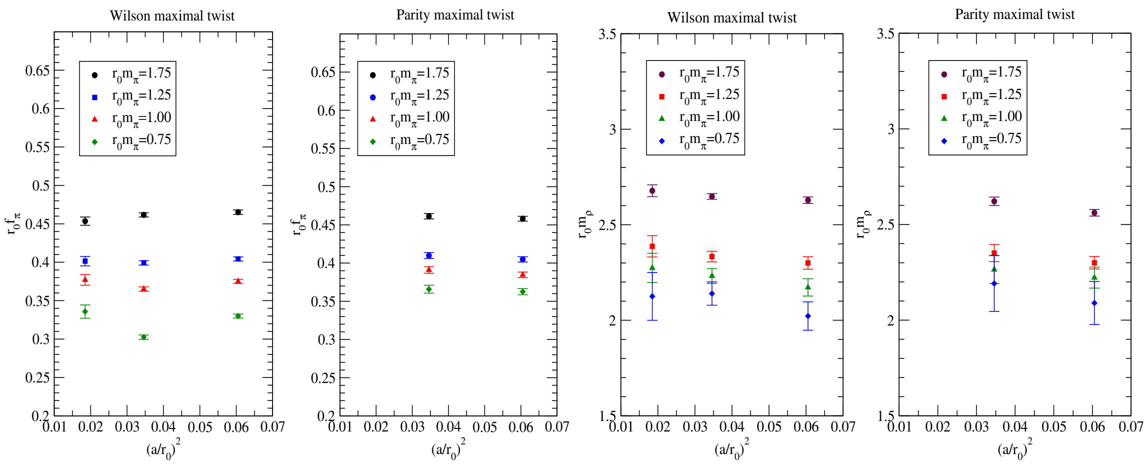

Figure 2: Scaling of the charged pion decay constant and the charged rho mass.

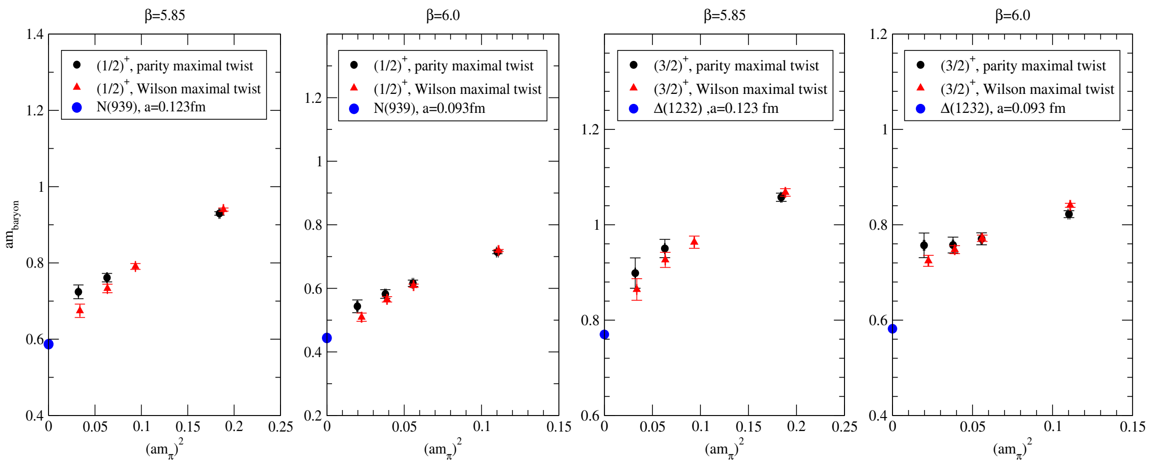

Figure 3: Baryon mass for $(\frac{1}{2})^+$ and $(\frac{3}{2})^+$ states as a function of the pion mass squared. The physical nucleon N(939) and $\Delta(1232)$ masses correspond to the lattice spacings in the legends.

5. Conclusions

Improved results for the hadron spectrum could be obtained from tmQCD and light quark masses. The parity definition of maximal twist was found to have a better scaling and chiral behaviour than the Wilson definition of maximal twist.

6. Acknowledgements

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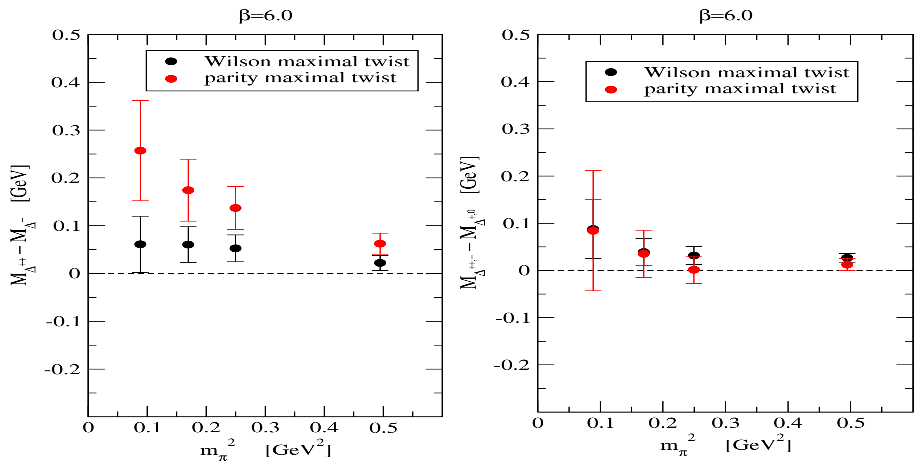

Figure 4: Δ mass splittings.

References

[1] [Alpha collaboration]R. Frezzotti, P. A. Grassi, S. Sint and P. Weisz, JHEP **0108** (2001) 058 [hep-lat/0101001].

[2] R. Frezzotti and G. C. Rossi, JHEP **0408**, 007 (2004) [hep-lat/0306014].

[3] [XLF Collaboration] K. Jansen, A. Shindler, C. Urbach and I. Wetzorke, Phys. Lett. B **586**, 432 (2004) [hep-lat/0312013].

[4] A. M. Abdel-Rehim and R. Lewis, Phys. Rev. D **71**, 014503 (2005) [hep-lat/0410047].

[5] [XLF Collaboration] W. Bietenholz *et al.*, JHEP **0412**, 044 (2004) [hep-lat/0411001].

[6] A. M. Abdel-Rehim, R. Lewis and R. M. Woloshyn, Phys. Rev. D **71**, 094505 (2005) [hep-lat/0503007].

[7] S. Aoki and O. Bär, Phys. Rev. D **70**, 116011 (2004) [hep-lat/0409006].

[8] S. R. Sharpe and J. M. S. Wu, Phys. Rev. D **70**, 094029 (2004) [hep-lat/0407025].

[9] R. Frezzotti, G. Martinelli, M. Papinutto and G. C. Rossi, [hep-lat/0503034].

[10] [XLF Collaboration], K. Jansen, M. Papinutto, A. Shindler, C. Urbach and I. Wetzorke, Phys. Lett. B **619**, 184 (2005) [hep-lat/0503031].

[11] F. Farchioni *et al.*, Eur. Phys. J. C **42**, 73 (2005) [hep-lat/0410031].

[12] S. R. Sharpe and J. M. S. Wu, Phys. Rev. D **71**, 074501 (2005) [hep-lat/0411021].

[13] S. R. Sharpe, [hep-lat/0509009].

[14] F. Farchioni *et al.*, [hep-lat/0509036].

[15] [XLF Collaboration] K. Jansen *et al.* , [hep-lat/0507032].

[16] Abdou M. Abdel-Rehim, Randy Lewis, and R. M. Woloshyn, [hep-lat/0509056].

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