

# Light-Quark FLIC Fermion Simulations of the $1^{-+}$ Exotic Meson

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We investigate the mass of the  $1^{-+}$  exotic meson, created with hybrid interpolating fields. Access to light quark masses approaching 25 MeV is facilitated by the use of the Fat-Link Irrelevant Clover (FLIC) fermion action, and large ( $20^3 \times 40$ ) lattices. Our results indicate that the  $1^{-+}$  exotic exhibits significant curvature close to the chiral limit, and yield a  $1^{-+}$  mass in agreement with the  $\pi_1(1600)$  candidate and exclusive of the  $\pi_1(1400)$ .

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## 1. Introduction

The exotic mesons comprise a rare vehicle for the elucidation of the relatively unexplored role of gluons in QCD. The Particle Data Group [1] reports two candidates for the  $1^{-+}$  exotic, the  $\pi_1(1400)$  at  $1.376(17)\text{GeV}$ , and the  $\pi_1(1600)$  at  $1.596_{-14}^{+25}\text{ GeV}$ . The interpretation of the experimental data continues to inspire discussion [2, 3].

Michael [11] provides a good summary of lattice results in this field up to 2003, concluding that the light-quark exotic is predicted by lattice studies to have a mass of  $1.9(2)\text{ GeV}$ , which differs from both experimental candidates. It should be emphasised, however, that previous results are derived from extrapolations from relatively heavy quark masses.

In order to minimize the need for extrapolation one requires access to quark masses near the chiral regime on large physical volumes. Our study considers a physical volume of  $(2.6\text{ fm})^3$ , and the  $\mathcal{O}(a)$ -improved FLIC fermion action [4, 5] whose improved chiral properties [6] permit the use of very light quark masses which are key to our results.

## 2. Lattice Simulations

We use local interpolating fields, coupling colour-octet quark bilinears to chromo-electric and chromo-magnetic fields. It is possible to generalise the interpolating fields to include non-local components where link paths are incorporated to maintain gauge invariance and carry the nontrivial quantum numbers of the gluon fields [7, 9]. Such an approach does not lead to an increase in signal for the ground state  $1^{-+}$  exotic commensurate with the increased computational cost of multiple fermion-matrix inversions.

Gauge-invariant Gaussian smearing [13, 14] is applied at the fermion source ( $t = 8$ ), and local sinks are used to maintain strong signal in the two-point correlation functions. In this work we considered four interpolating fields for the  $1^{-+}$  exotic:

$$\chi_1 = \bar{q}^a \gamma_4 E_j^{ab} q^b, \quad (2.1)$$

$$\chi_2 = i \varepsilon_{jkl} \bar{q}^a \gamma_k B_l^{ab} q^b, \quad (2.2)$$

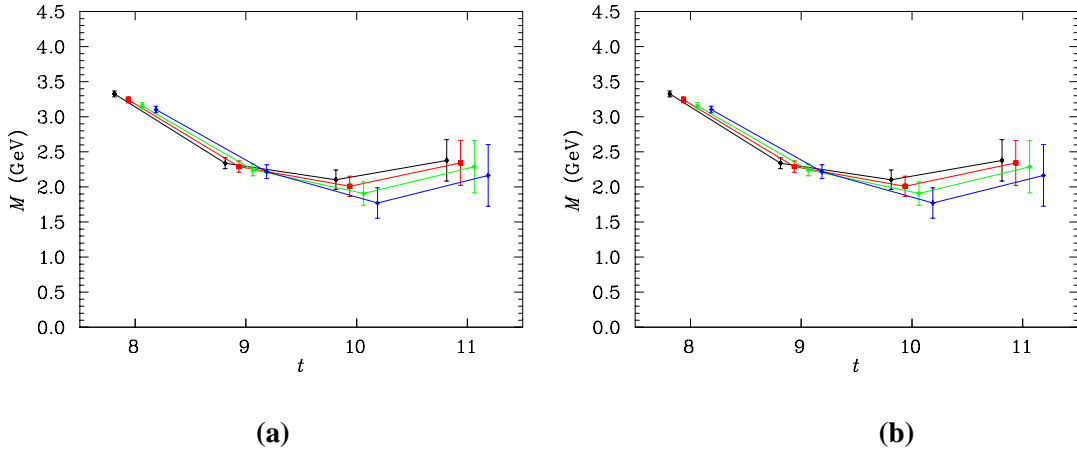
$$\chi_3 = i \varepsilon_{jkl} \bar{q}^a \gamma_4 \gamma_k B_l^{ab} q^b, \quad (2.3)$$

and

$$\chi_4 = \varepsilon_{jkl} \bar{q}^a \gamma_5 \gamma_4 \gamma_k E_l^{ab} q^b. \quad (2.4)$$

The interpolating fields which couple large-large and small-small spinor components (i.e  $\chi_2$  and  $\chi_3$ ) provide the strongest signal for the  $1^{-+}$  state.

In order to obtain the chromo-electric and chromo-magnetic fields with which we build the hybrid operators, we make use of a modified version of APE smearing [15], in which the smeared links do not involve averages which include links in the temporal direction. In this way we preserve the notion of a Euclidean ‘time’ and avoid overlap of the creation and annihilation operators. In this study, the smearing fraction  $\alpha = 0.7$  (keeping 0.3 of the original link) and the process of smearing and  $SU(3)$  link projection is iterated four times [17]. Smearing the links permits the use of highly improved definitions of the lattice field strength tensor, from which our hybrid operators are derived. Details of the  $\mathcal{O}(a^4)$ -improved tensor are given in [16].

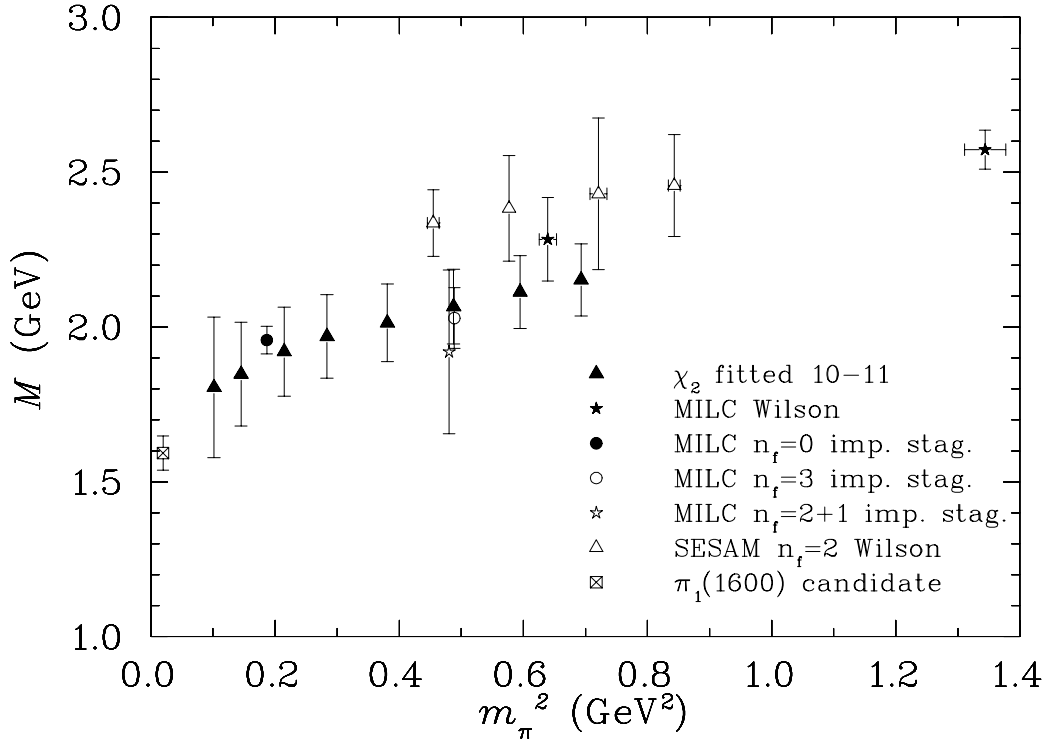


**Figure 1:** Effective masses extracted with interpolators  $\chi_2$  (a) and  $\chi_3$  (b).

Propagators are generated using the fat-link irrelevant clover (FLIC) fermion action [5] where the irrelevant Wilson and clover terms of the fermion action are constructed using APE-Smeared links [15], while the relevant operators use the untouched (thin) gauge links. In the FLIC action, this yields improved chiral properties and reduces the problem of exceptional configurations encountered with clover actions [6], and minimizes the effect of renormalization on the action improvement terms [18]. Details of this approach may be found in reference [5]. FLIC fermions provide a new form of nonperturbative  $\mathcal{O}(a)$  improvement [6, 18] where near-continuum results are obtained at finite lattice spacing.

We use quenched-QCD gauge fields created by the CSSM Lattice Collaboration with the  $\mathcal{O}(a^2)$  mean-field improved Lüscher-Weisz plaquette plus rectangle gauge action [19] using the plaquette measure for the mean link. The CSSM configurations are generated using the Cabibbo-Marinari pseudo-heat-bath algorithm [20] using a parallel algorithm with appropriate link partitioning [21]. To improve the ergodicity of the Markov chain process, the three diagonal SU(2) subgroups of SU(3) are looped over twice [22] and a parity transformation [23] is applied randomly to each gauge field configuration saved during the Markov chain process.

The calculations of meson masses are performed on  $20^3 \times 40$  lattices at  $\beta = 4.53$ , which provides a lattice spacing of  $a = 0.128(2)$  fm set by the Sommer parameter  $r_0 = 0.49$  fm. A fixed boundary condition in the time direction is used for the fermions by setting  $U_t(\vec{x}, N_t) = 0 \forall \vec{x}$  in the hopping terms of the fermion action, with periodic boundary conditions imposed in the spatial directions. Eight quark masses are considered in the calculations and the strange quark mass is taken to be the third heaviest quark mass. This provides a pseudoscalar mass of 697 MeV which compares well with the experimental value of  $(2M_K^2 - M_\pi^2)^{1/2} = 693$  MeV motivated by leading order chiral perturbation theory. The analysis is based on a sample of 345 configurations, and the error analysis is performed by a third-order single-elimination jackknife, with the  $\chi^2$  per degree of freedom ( $\chi^2/dof$ ) obtained via covariance matrix fits.



**Figure 2:** A survey of results in this field. Open and closed symbols denote dynamical and quenched simulations respectively. The MILC results are taken from [8] and show their  $Q^4, 1^{-+} \rightarrow 1^{-+}$  results, fitted from  $t = 3$  to  $t = 11$ .

**Table 1:**  $1^{-+}$  Exotic Meson mass (GeV) vs square of pion mass ( $\text{GeV}^2$ ).

$m_\pi^2$	$\chi_2$ fit 10-11		$\chi_2$ fit 10-12		$\chi_3$ fit 10-11	
	$m$	$\chi^2/dof$	$m$	$\chi^2/dof$	$m$	$\chi^2/dof$
0.693(3)	2.15(12)	0.69	2.16(11)	0.44	2.20(15)	0.45
0.595(4)	2.11(12)	0.77	2.12(11)	0.51	2.18(16)	0.46
0.488(3)	2.07(12)	0.85	2.08(12)	0.59	2.15(17)	0.41
0.381(3)	2.01(12)	0.91	2.03(12)	0.65	2.14(19)	0.29
0.284(3)	1.97(13)	0.78	1.98(13)	0.55	2.27(29)	0.0001
0.215(3)	1.92(14)	0.78	1.92(14)	0.40	2.25(31)	0.02
0.145(3)	1.85(17)	0.57	1.84(17)	1.76	2.26(37)	0.02
0.102(4)	1.80(23)	0.13	1.75(23)	3.04	2.46(58)	0.03

### 3. Results

Figure 1 shows the effective mass for the two preferred interpolators. For clarity, we have plotted the results for every second quark mass used in our simulation. The effective masses exhibit plateaus at 0.256 fm from the source which is consistent with Ref. [8], where a similar effect is seen after approximately 0.21 to 0.28 fm.

Table 1 summarizes our results for the mass of the  $1^{-+}$  meson, with the squared pion-mass

provided as a measure of the input quark mass. The agreement between the interpolators is significant, as we expect them to possess considerably different excited-state contributions, based on experience with pseudoscalar interpolators [24].

Fig. 2 summarizes a collection of results for the mass of the  $1^{-+}$  obtained in lattice QCD simulations thus far. The current results presented herein (full triangles) are compared with results from the MILC [8, 10] and SESAM [9] collaborations, both of which provide a consistent scale via  $r_0$ .

We perform a linear fit to the  $1^{-+}$  mass using the four lightest quark masses and a quadratic form to all 8 masses. Systematic uncertainties associated with chiral nonanalytic curvature are estimated at 50 MeV [26, 25]. A third-order single-elimination jackknife error analysis yields masses of 1.74(24) and 1.74(25) GeV for the linear and quadratic fits, respectively. These results agree within one standard deviation with the experimental  $\pi_1(1600)$  result of  $1.596^{+25}_{-14}$  GeV, and exclude the mass of the  $\pi_1(1400)$  candidate.

#### 4. Conclusion

We have found a compelling signal for the  $J^{PC} = 1^{-+}$  exotic meson at very light quark masses, from which we can extrapolate a physical mass of 1.74(24) GeV. Thus for the first time in lattice studies, we find a  $1^{-+}$  mass in agreement with the  $\pi_1(1600)$  candidate.

Looking forward, it will be important to quantify the effects of the quenched approximation. Of particular interest will be the extent to which the curvature observed in approaching the chiral regime is preserved in full QCD.

Whilst the rapidity with which we establish a plateau in our effective mass plots suggests that our current fermion operator smearing is near optimal for isolating the ground state, it might be possible to reduce the statistical errors through a careful selection of parameters coming out of a systematic exploration of the parameter space.

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