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Brane Condensation and Confinement

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We study the static quantum potential for a theory of anti-symmetric tensor fields that results from the condensation of topological defects, within the framework of the gauge-invariant but path-dependent variables formalism. Our calculations show that the interaction energy is the sum of a Yukawa and a linear potentials, leading to the confinement of static probe charges.

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1. Introduction

One of the fundamental issues of theoretical physics is that of the confinement for the fundamental constituents of matter. In fact distinction between the apparently related phenomena of screening and confinement is of considerable importance in our present understanding of gauge theories. Field theories that yield the linear potential are important to particle physics, since those theories may be used to describe the confinement of quarks and gluons and be considered as effective theories of quantum chromodynamics.

We study the confinement versus screening properties of some theories of massless antisymmetric tensors, magnetically and electrically coupled to topological defects that eventually condense, as a consequence of the Julia–Toulouse mechanism (JTM)[1]. This mechanism is the dual to the Higgs mechanism and has been shown to lead to a concrete massive antisymmetric theory with a jump of rank. We show that in the presence of two tensor fields the condensation induces not only a mass term and a jump of rank but also a BF coupling which will be responsible for the change from the screening to the confining phase of the theory.

An important issue here is the nature of the phase transition in the presence of a finite condensate of topological defects. It is this aspect, in $D = d+1$ dimensions for generic antisymmetric tensors theories, that is of importance for us. This issue was discussed long time ago [1] in the framework of ordered solid-state media and more recently in the relativistic context [2]. The basic idea in Ref.[1] was to consider models with non-trivial homotopy group able to support stable topological defects characterized by a length scale $r = 1/M$, where the mass parameter M is a cut-off for the low-energy effective field theory. The long wavelength fluctuations of the continuous distribution of topological defects are the new hydrodynamical modes for the effective theory that appear when topological defects condense. In [1] there is an algorithm to identify these modes in the framework of ordered solid-state media. However, due to the presence of non-linear terms, the lack of relativistic invariance and the need to introduce dissipation terms it becomes difficult to write down an action for the phase with a condensate of topological defects. In the relativistic context none of the above problems is present. In [2] an explicit form for the action in the finite condensate phase, for generic compact antisymmetric field theories was found. In this context the JTM is the natural generalization of the confinement phase for a vector gauge field.

In this paper we make use of the JTM, as presented in [2], to study the low-energy field theory of a pair of massless anti-symmetric tensor fields, say A_p and B_q with $p+q+2 = D$, coupled electrically and magnetically to a large set of $(q-1)$-branes, characterized by charge e and a Chern-Kernel Λ_{p+1} [3], that eventually condense. It is shown that the effective theory that results displays the confinement property by computing explicitly the effective potential for a pair of static, very massive point probes. Basically, we are interested in studying the JTM in model field theories involving B_q and A_p coupled to a $(q-1)$-brane, according to the following action

$$\mathcal{S} = \int \frac{1}{2}\frac{(-1)^q}{(q+1)!}\left[H_{q+1}(B_q)\right]^2 + e B_q J^q(\Lambda) + \frac{1}{2}\frac{(-1)^p}{(p+1)!}\left[F_{p+1}(A_p) - e\Lambda_{p+1}\right]^2 \tag{1.1}$$

and consider the condensation phenomenon when Λ_{p+1} becomes the new massive mode of the effective theory. Our compact notation here goes as follows. The field strength reads $F_{p+1}(A_p) = F_{\mu_1\mu_2\cdots\mu_{p+1}} = \partial_{[\mu_1}A_{\mu_2\cdots\mu_{p+1}]}$, $H_{q+1}(B_q) = \partial_{[\mu_1}B_{\mu_2\cdots\mu_{q+1}]}$ and $\Lambda_{p+1} = \Lambda_{\mu_1\cdots\mu_{p+1}}$ is a totally anti-symmetric object of rank $(p+1)$. The conserved current $J^q(\Lambda)$ is given by a delta-function over the world-volume of the $(q-1)$-brane [4]. This conserved current may be rewritten in terms of the kernel Λ_{p+1} as

$$J^q(\Lambda) = \frac{1}{(p+1)!}\varepsilon^{q,\alpha,p+1}\partial_\alpha \Lambda_{p+1} \; , \tag{1.2}$$

and $\varepsilon^{q,\alpha,p+1} = \varepsilon^{\mu_1\ldots\mu_q,\alpha,\nu_1\ldots\nu_{p+1}}$. This notation will be used in the discussion of the JTM in the next section as long as no chance of confusion occurs.

2. The Julia–Toulouse mechanism and the action in the condensed phase

Although the JTM becomes problematic in the ordered solid state media, Quevedo and Trugenberg [2] have shown that it leads to simple demands in the study of compact antisymmetric tensor, where it produces naturally the effective action for the new phase. They observed that when the $(d-h-1)$-branes condense this generates a new scale Δ related to the average density ρ of intersection points of the $(d-h)$-dimensional world-hypersurfaces of the condensed branes with any $(h+1)$-dimensional hyperplane. The four requirements to describe effectively the dense phase are: (i) an action built up to two derivatives in the new field possessing (ii) gauge invariance, (iii) relativistic invariance and, most important, (iv) the need to recover the original model in the limit $\Delta \to 0$. One is therefore led to consider the action for the condensate as

$$\mathcal{S}_\Omega = \int \frac{(-1)^h}{2\Delta^2 (h+1)!}\left[F_{h+1}(\Omega_h)\right]^2 - \frac{(-1)^h\, h!}{2\, e^2}\left[\Omega_h - H_h(\phi_{h-1})\right]^2 \tag{2.1}$$

where $H_{\mu_1\cdots\mu_h} = \partial_{[\mu_1}\phi_{\mu_2\cdots\mu_h]}$ and the underlying gauge invariance is manifest by the simultaneous transformations $\Omega_{\mu_1\cdots\mu_h} \to \Omega_{\mu_1\cdots\mu_h} + \partial_{[\mu_1}\psi_{\mu_2\cdots\mu_h]}$ and $\phi_{\mu_1\cdots\mu_{h-1}} \to \phi_{\mu_1\cdots\mu_{h-1}} + \psi_{\mu_1\cdots\mu_{h-1}}$. Upon fixing this invariance one can drop all considerations over ϕ_{h-1} after absorbing $H_h(\phi_{h-1})$ into Ω_h, so that the action describes the exact number of degrees of freedom of a massive field whose mass parameter reads $m = \Delta/e$. This process, named as JTM, is dual to the Higgs mechanism. Here on the other hand, the new modes generated by the condensation of magnetic topological defects absorbs the original variables of the effective theory, thereby acquiring a mass while in the Higgs mechanism it is the original field that incorporates the degrees of freedom of the electric condensate to acquire mass. This difference explains the change of rank in the JT mechanism that is not present in the Higgs process. In the limit $\Delta \to 0$ the only relevant field configurations are those that satisfy the constraint $F_{h+1}(\Omega_h) = 0$ whose solution reads $\Omega_{\mu_1\cdots\mu_h} = \partial_{[\mu_1}\psi_{\mu_2\cdots\mu_h]}$ where ψ_{h-1} is an $(h-1)$-anti-symmetric tensor field. The field ψ_{h-1} can then be absorbed into ϕ_{h-1} this way recovering the original low-energy effective action before condensation.

The distinctive feature of the JT mechanism is that after condensation Λ_{p+1} is elevated to the condition of propagating field. The new degree of freedom absorbs the degrees of freedom of the tensor A_p this way completing its longitudinal sector. The new mode is therefore explicitly massive. Since $A_p \to \Lambda_{p+1}$ there is a change of rank with dramatic consequences. The last term in (1.1), displaying the magnetic coupling between the field-tensor $F_{p+1}(A_p)$ and the $(q-1)$-brane, becomes the mass term for the new effective theory in terms of the tensor field Λ_{p+1} and a new dynamical term is induced by the condensation. The minimal coupling of the B_q tensor becomes responsible for another contribution for the mass, this time of topological nature. Indeed the second term (1.1) becomes a "$B\wedge F(\Lambda)$" term between the remaining propagating modes, inducing topological mass, in addition to the induced condensed mass,

$$\begin{aligned} \mathcal{S}_{cond} &= \int \frac{(-1)^q}{2(q+1)!}\left[H_{q+1}(B_q)\right]^2 + e\,B_q\varepsilon^{q,\alpha,p+1}\partial_\alpha\Lambda_{p+1} + \\ &+ \int \frac{(-1)^{p+1}}{2(p+2)!}\left[F_{p+2}(\Lambda_{p+1})\right]^2 - \frac{(-1)^{p+1}(p+1)!}{2}m^2\Lambda_{p+1}^2. \end{aligned} \tag{2.2}$$

Recall that the theory before condensation displayed two independent fields coupled to a $(q-1)$–brane. The nature of the two couplings were however different. The A_p tensor, that was magnetically coupled to the brane, was then absorbed by the condensate after phase transition. On the other hand, the electric coupling, displayed by the B_q tensor, became a "$B\wedge F(\Lambda)$" topological term after condensation. There has been a drastic change in the physical scenario. We want next to obtain an effective action for the B_q tensor. To this end we shall next integrate out the field Λ describing the condensate to obtain, our final effective theory as

$$\mathcal{S}_{eff} = \int \frac{(-1)^{q+1}}{2\,(q+1)!}H_{q+1}(B_q)\left(1+\frac{e^2}{\Delta^2+m^2}\right)H^{q+1}(B_q). \tag{2.3}$$

3. Interaction energy

Next we examine the screening versus confinement issue and consider an example involving two Maxwell tensors coupled electrically and magnetically to a point-charge. After condensation we end up with a Maxwell and a massive Kalb-Ramond field coupled topologically to each other. We shall calculate the interaction energy for the effective theory between external probe sources by computing the expectation value of the energy operator H in the physical state $|\Phi\rangle$ describing the sources, denote by $\langle H\rangle_\Phi$. The Kalb-Ramond field $\Lambda_{\mu\nu}$ carrying the degrees of freedom of the condensate is integrate out leading to

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}\left(1+\frac{e^2}{\triangle^2+m^2}\right)F^{\mu\nu} - A_0J^0, \tag{3.1}$$

where J^0 is an external current. We observe that the limits $e\rightarrow 0$ or $m\rightarrow 0$ are well defined and lead to a pure Maxwell theory or to a topologically massive model. Since the probe charges only couple to the Maxwell fields, the Kalb- Ramond condensate will not contribute to their interaction energy if the parameter $e\rightarrow 0$ since the Maxwell field and the condensate decouple. The second limit means that we are back to the dilute phase. As so the confinement disappears being taken over by an screening phase controlled by the parameter e playing the role of topological mass.

The canonical quantization of this theory from the Hamiltonian point of view follows straightforwardly. The canonical momenta read $\Pi^\mu = -\left(1+\frac{e^2}{\Delta^2+m^2}\right)F^{0\mu}$ with the only nonvanishing canonical Poisson brackets being $\left\{A_\mu(t,x),\Pi^\nu(t,y)\right\} = \delta_\mu^\nu\delta(x-y)$. Since Π_0 vanishes we have the usual primary constraint $\Pi_0=0$, and $\Pi^i = \left(1+\frac{e^2}{\Delta^2+m^2}\right)F^{i0}$. The canonical Hamiltonian is thus

$$H_C = \int d^3x\left\{-\frac{1}{2}\Pi^i\left(1+\frac{e^2}{\Delta^2+m^2}\right)^{-1}\Pi_i + \Pi^i\partial_iA_0 + \frac{1}{4}F_{ij}\left(1+\frac{e^2}{\Delta^2+m^2}\right)F^{ij} + A_0J^0\right\}. \tag{3.2}$$

Conservation of the primary constraint leads to the Gauss-law $\Gamma_1(x) \equiv \partial_i \Pi^i - J^0 = 0$. The preservation of Γ_1 for all times does not give rise to any further constraints. The theory possess only two first class constraints being therefore gauge-invariant. The extended Hamiltonian that generates translations in time then reads $H = H_C + \int d^3x\,(c_0(x)\,\Pi_0(x) + c_1(x)\,\Gamma_1(x))$, where $c_0(x)$ and $c_1(x)$ are the Lagrange multiplier fields. Moreover, it is straightforward to see that $\dot{A}_0(x) = [A_0(x), H] = c_0(x)$, which is an arbitrary function. Since $\Pi^0 = 0$ always, neither A^0 nor Π^0 are of any interest

The quantization of the theory requires the removal of nonphysical variables, which is done by imposing a gauge condition such that the full set of constraints becomes second class. A convenient choice is found to be [8] $\Gamma_2(x) \equiv \int\limits_{C_{\xi x}} dz^\nu A_\nu(z) \equiv \int\limits_0^1 d\lambda x^i A_i(\lambda x) = 0$, where λ $(0 \leq \lambda \leq 1)$ is the parameter describing the spacelike straight path $x^i = \xi^i + \lambda(x-\xi)^i$, and ξ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. In this case, the only nonvanishing equal-time Dirac bracket is

$$\left\{A_i(x), \Pi^j(y)\right\}^* = \delta_i^j \delta^{(3)}(x-y) - \partial_i^x \int\limits_0^1 d\lambda x^j \delta^{(3)}(\lambda x - y). \tag{3.3}$$

We now turn to the problem of obtaining the interaction energy between pointlike sources in the model under consideration. The state $|\Phi\rangle$ representing the sources is obtained by operating over the vacuum with creation/annihilation operators. We want to stress that, by construction, such states are gauge invariant. In the case at hand we consider the gauge-invariant stringy $\left|\overline{\Psi}(\mathbf{y})\Psi(\mathbf{y}')\right\rangle$, where a fermion is localized at $\mathbf{y}\prime$ and an antifermion at $\mathbf{y}$ as follows [9],

$$|\Phi\rangle \equiv \left|\overline{\Psi}(\mathbf{y})\Psi(\mathbf{y}\prime)\right\rangle = \overline{\psi}(\mathbf{y}) \exp\left(iq\int\limits_{\mathbf{y}\prime}^{\mathbf{y}} dz^i A_i(z)\right)\psi(\mathbf{y}\prime)\,|0\rangle, \tag{3.4}$$

where $|0\rangle$ is the physical vacuum state and the line integral appearing in the above expression is along a spacelike path starting at $\mathbf{y}\prime$ and ending $\mathbf{y}$, on a fixed time slice. It is worth noting here that the strings between fermions have been introduced in order to have a gauge-invariant function $|\Phi\rangle$. In other terms, each of these states represents a fermion-antifermion pair surrounded by a cloud of gauge fields sufficient to maintain gauge invariance. As we have already indicated, the fermions are taken to be infinitely massive (static).

From our above discussion, we see that $\langle H\rangle_\Phi$ reads

$$\langle H\rangle_\Phi = \langle\Phi|\int d^3\left\{-\frac{1}{2}\Pi_i\left(1 - \frac{e^2}{\nabla^2 - m^2}\right)^{-1}\Pi^i\right\}|\Phi\rangle, \tag{3.5}$$

where, in this static case, $\Delta^2 = -\nabla^2$. Observe that when $e = 0$ we obtain the pure Maxwell theory, as mentioned after (3.1). From now on we will suppose $e \neq 0$.

Next, from the foregoing Hamiltonian analysis, $\langle H\rangle_\Phi$ becomes $\langle H\rangle_\Phi = \langle H\rangle_0 + V^{(1)} + V^{(2)}$, where $\langle H\rangle_0 = \langle 0|H|0\rangle$. The $V^{(1)}$ and $V^{(2)}$ terms are given by:

$$V^{(1)} = -\frac{q^2}{2}\int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i' \delta^{(3)}\left(x - z'\right)\frac{1}{\nabla_x^2 - M^2}\nabla_x^2 \int_{\mathbf{y}}^{\mathbf{y}'} dz^i \delta^{(3)}(x-z), \tag{3.6}$$

and

$$V^{(2)}=\frac{q^2m^2}{2}\int d^3x\int_{\mathbf{y}}^{\mathbf{y}'}dz_i'\delta^{(3)}\left(x-z'\right)\frac{1}{\nabla_x^2-M^2}\int_{\mathbf{y}}^{\mathbf{y}'}dz^i\delta^{(3)}\left(x-z\right),\tag{3.7}$$

where $M^2\equiv m^2+e^2$ and the integrals over z^i and z_i' are zero except on the contour of integration.

The $V^{(1)}$ term may look peculiar, but it is nothing but the familiar Yukawa interaction plus self-energy terms. In effect, as was explained in Ref. [10], the expression (3.6) can also be written as

$$V^{(1)}=\frac{e^2}{2}\int_{\mathbf{y}}^{\mathbf{y}'}dz_i'\partial_i^{z'}\int_{\mathbf{y}}^{\mathbf{y}'}dz^i\partial_z^iG\left(\mathbf{z}',\mathbf{z}\right)=-\frac{q^2}{4\pi}\frac{e^{-M|\mathbf{y}-\mathbf{y}'|}}{|\mathbf{y}-\mathbf{y}'|},\tag{3.8}$$

where we used that the Green function $G(\mathbf{z}',\mathbf{z})=\frac{1}{4\pi}\frac{e^{-M|\mathbf{z}'-\mathbf{z}|}}{|\mathbf{z}'-\mathbf{z}|}$ and remembered that the integrals over z^i and z_i' are zero except on the contour of integration. The expression then reduces to the Yukawa-type potential after subtracting the self-energy terms.

We now turn our attention to the calculation of the $V^{(2)}$ term, which is given by

$$V^{(2)}=\frac{q^2m^2}{2}\int_{\mathbf{y}}^{\mathbf{y}'}dz'^i\int_{\mathbf{y}}^{\mathbf{y}'}dz^iG(\mathbf{z}',\mathbf{z}).\tag{3.9}$$

It is appropriate to observe here that the above term is similar to the one found for the system consisting of a gauge field interacting with a massive axion field [10]. Notwithstanding, in order to put our discussion into context it is useful to summarize the relevant aspects of the calculation described previously [10]. In effect, as was explained in Ref. [10], by using the Green function in momentum space, that is, $\frac{1}{4\pi}\frac{e^{-M|\mathbf{z}'-\mathbf{z}|}}{|\mathbf{z}'-\mathbf{z}|}=\int\frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot(\mathbf{z}'-\mathbf{z})}}{\mathbf{k}^2+M^2}$, the expression (3.9) can also be written as

$$V^{(2)}=q^2m^2\int\frac{d^3k}{(2\pi)^3}\left[1-\cos\left(\mathbf{k}\cdot\mathbf{r}\right)\right]\frac{1}{(\mathbf{k}^2+M^2)}\frac{1}{(\hat{\mathbf{n}}\cdot\mathbf{k})^2},\tag{3.10}$$

where $\hat{\mathbf{n}}\equiv\frac{\mathbf{y}-\mathbf{y}'}{|\mathbf{y}-\mathbf{y}'|}$ is a unit vector and $\mathbf{r}=\mathbf{y}-\mathbf{y}'$ is the relative vector between the quark and antiquark. Since $\hat{\mathbf{n}}$ and $\mathbf{r}$ are parallel, we get accordingly $V^{(2)}=\frac{q^2m^2}{8\pi^3}\int\limits_{-\infty}^{\infty}\frac{dk_r}{k_r^2}\left[1-\cos\left(k_rr\right)\right]\int\limits_0^{\infty}d^2k_T\frac{1}{(k_r^2+k_T^2+M^2)}$, where k_T denotes the momentum component perpendicular to $\mathbf{r}$. Integration over k_T yields $V^{(2)}=\frac{q^2m^2}{8\pi^2}\int\limits_{-\infty}^{\infty}\frac{dk_r}{k_r^2}\left[1-\cos\left(k_rr\right)\right]\ln\left(1+\frac{\Lambda^2}{k_r^2+M^2}\right)$, where Λ is an ultraviolet cutoff. We also observe at this stage that similar integral was obtained independently in Ref.[11] in the context of the dual Ginzburg-Landau theory by an entirely different approach.

We now proceed to compute the previous integral. For this purpose we introduce a new auxiliary parameter ε by making in the denominator of the previous integral the substitution $k_r^2\to k_r^2+\varepsilon^2$. Thus it follows that $V^{(2)}\equiv\lim_{\varepsilon\to0}\widetilde{V}^{(2)}=\lim_{\varepsilon\to0}\frac{q^2m^2}{8\pi^2}\int\limits_{-\infty}^{\infty}\frac{dk_r}{(k_r^2+\varepsilon^2)}\left[1-\cos\left(k_rr\right)\right]\ln\left(1+\frac{\Lambda^2}{k_r^2+M^2}\right)$. We further note that the integration on the k_r-complex plane yields $\widetilde{V}^{(2)}=\frac{q^2m^2}{8\pi}\left(\frac{1-e^{-\varepsilon|\mathbf{y}-\mathbf{y}'|}}{\varepsilon}\right)\ln\left(1+\frac{\Lambda^2}{M^2-\varepsilon^2}\right)$. Taking the limit $\varepsilon\to0$, this expression then becomes $V^{(2)}=\frac{q^2m^2}{8\pi}|\mathbf{y}-\mathbf{y}'|\ln\left(1+\frac{\Lambda^2}{M^2}\right)$.

This, together with Eq.(3.8), immediately shows that the potential for two opposite charges located at $\mathbf{y}$ and $\mathbf{y}'$ is given by

$$V(L)=-\frac{q^2}{4\pi}\frac{e^{-ML}}{L}+\frac{q^2m^2}{8\pi}L\ln\left(1+\frac{\Lambda^2}{M^2}\right),\tag{3.11}$$

where $L\equiv|\mathbf{y}-\mathbf{y}'|$.

4. Final remarks

We have studied the confinement versus screening issue for a pair of antisymmetric tensors coupled to topological defects that eventually condense, giving a specific realization of the Julia–Toulouse phenomenon. We have seen that the Julia–Toulouse mechanism for a couple of massless antisymmetric tensors is responsible for the appearance of mass and the jump of rank in the magnetic sector while the electric sector becomes a BF–type coupling. The condensate absorbs and replaces one of the tensors and becomes the new massive propagating mode but does not couple directly to the probe charges. The effects of the condensation are however felt through the BF coupling with the remaining massless tensor. It is therefore not surprising that they become manifest in the interaction energy for the effective theory. We have obtained the effective theory for the condensed phase in general and computed the interaction energy between two static probe charges, in a specific example, in order to test the confinement versus screening properties of the effective model. Our results show that the interaction energy in fact contains a linear confining term and an Yukawa type potential. It can be observed that confinement completely disappears in the limit $m \to 0$ while the screening takes over controlled by the topological mass parameter instead. Although we have considered the case where the effective model consists of the BF–coupling between a Kalb-Ramond field (that represents the condensate) and a Maxwell field, our results seem to be quite general. A direct calculation for tensors of arbitrary rank in the present approach is however a quite challenging problem that we hope to be able to report in the future.

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