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General Unitary TFD Formulation for Superstrings

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A generalization of the Thermo Field Dynamics (TFD) for fermionic degrees of freedom is proposed. Such a generalization follows a previous one where the SU(1,1) thermal group was used to obtain the closed bosonic string at finite temperature. The SU(2) thermal group is introduced to construct a general thermal Bogoliubov transformation to get the type IIB superstring at finite temperature.

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Thermo Field Dynamics (TFD) [1] is a real time approach to deal with systems at finite temperature. Its basic elements are the doubling of degrees of freedom of the system under study and a (thermal) Bogoliubov transformation for entangle such a duplicated degrees. With the doubling, one obtains an enlarged Hilbert space composed by the original and an auxiliary space, identical to the first and related to the so called tilde system. The enlarged Hilbert space is denoted by a hat and is given by $\widehat{\mathcal{H}} = \mathcal{H} \otimes \widetilde{\mathcal{H}}$. The original and tilde systems are related by a mapping called tilde conjugation rules. These rules are related with the Tomita-Takesaki modular operator of the statistical mechanics algebraic approach [2, 3].

The thermal Bogoliubov transformation is obtained using a generator in such a way that, in a finite volume limit, the transformation is unitary and preserves the tilde conjugation rules. The thermal effects arise from the vacuum correlation introduced by the transformation over the enlarged system vacuum.

The construction above outlined was the first one for the TFD. However, one can find a set of generator that maintains the thermal nature of the transformation. The set of generators is shown to be a linear combination of operators that forms an oscillator representation of $SU(1,1)$ group for bosons and $SU(2)$ for fermions [4, 5]. These sets for fermionic and bosonic systems can be constructed in two different manner, providing two possible generalizations of the TFD approach. In one case the tilde conjugation rules are preserved but the transformation, in a finite volume limit, is non-unitary. This construction was largely developed as one can see, for example in Ref. [4]. In the second case the transformation is unitary, in a finite volume limit, but the tilde conjugation rules are not preserved.

The unitary case was applied to bosonic string and D_p-brane in Ref. [6]. In a recent paper [7] the general unitary $SU(1,1)$ TFD formulation was proposed for bosonic systems. Such a formulation considers a transformed Tomita-Takesaki modular operator, once that it does not commute with the general generator of the thermal transformation [8]. As a consequence, the tilde conjugation rules were redefined, in the transformed space, and called breve conjugation rules. The generalized $SU(1,1)$ thermal vacuum is invariant under breve conjugation.

The objective of this work is to extend such a unitary TFD formulation to include fermionic degrees of freedom, using as target system the type IIB Green-Schwarz superstring. Here, a TFD construction taking into account explicitly the level-matching condition is applied [9, 10]. We consider the light-cone coordinates $X^{\pm} = \frac{1}{\sqrt{2}}\left(X^9 \pm X^0\right)$ and we write the remaining 8 components of the spinors (after Kappa symmetry fixing) as S^a, $\bar{S}^a$, composing the $\mathbf{8_s}$ representation of $SO(8)$. The chiral representation of $SO(8)$ gamma matrices is used.

The solutions of the equation of motion for the system can be expanded in string modes. After the quantization we choose the oscillator description of such a modes in order to deal with the standard commutation and anti-commutation relations of the harmonic oscillator. The left-moving bosonic and fermionic oscillators are denoted by a_n^I, $a_n^{\dagger I}$ and S_n^a, $S_n^{\dagger a}$. The right-moving modes will be denoted by a "bar" over the operators. In general the vacuum of the fermionic zero modes are chosen to be a set of states denoted by $|I\rangle, |\dot{a}\rangle$, satisfying: $S_0^a |I\rangle = \gamma^I_{a\dot{a}} |\dot{a}\rangle$ and $S_0^a |\dot{a}\rangle = \gamma^I_{a\dot{a}} |I\rangle$ [11]. In order to apply the TFD algorithm it is necessary to use a different fermionic vacuum than the usual one. The vacuum is chosen to be a state $|0,p^+\rangle$ such that

$$S^a \left|0,p^+\right\rangle = 0,$$

$$S_n^a\left|0,p^+\right\rangle = \bar{S}_n^a\left|0,p^+\right\rangle = 0, \qquad n>0,$$
$$a_n^I\left|0,p^+\right\rangle = \bar{a}_n^I\left|0,p^+\right\rangle = 0, \qquad n>0, \tag{1}$$

with the following definition for the fermionic zero modes creation-annihilation operators:

$$S^a = \frac{1}{\sqrt{2}}\left(S_0^a + i\bar{S}_0^b\right), \qquad S^{\dagger a} = \frac{1}{\sqrt{2}}\left(S_0^a - i\bar{S}_0^a\right), \tag{2}$$

which satisfies

$$\left\{S^a, S^{\dagger b}\right\} = \delta^{ab}, \qquad \left\{S^a, S^b\right\} = \left\{S^{\dagger a}, S^{\dagger b}\right\} = 0. \tag{3}$$

The light-cone Hamiltonian is calculated in a standard way and it is written as

$$p_+H = \frac{p^ip^i}{2} + \sum_{n=1} n\left(a_n^{\dagger I}a_n^I + \bar{a}_n^{\dagger I}\bar{a}_n + S_n^{\dagger a}S_n^a + \bar{S}_n^{\dagger a}\bar{S}_n^a\right). \tag{4}$$

The 32 supersymmetries are divided in a set of 16 kinematical supercharges, that belong to $\mathbf{8_s}$ of $SO(8)$, and 16 dynamical supercharges, that transform the fields of the same supermultiplet and belong to $\mathbf{8_c}$ of $SO(8)$. The kinematical and dynamical supercharges can be written as $Q_a^{\pm} = Q_a \pm i\bar{Q}_a$ and $Q_{\dot{a}}^{\pm} = Q_{\dot{a}} \pm i\bar{Q}_{\dot{a}}$, respectively, where

$$\sqrt{p^+}Q_a = S_0^a,$$
$$\sqrt{p^+}Q_{\dot{a}} = P_0^I\left(\gamma^I S_0\right)_{\dot{a}} + \sum_{n=1}\left[\sqrt{2\omega_n}\left(a_n^{\dagger I}\gamma^I S_n + a_n^I\gamma^I S_n^{\dagger}\right)_{\dot{a}}\right], \tag{5}$$

and $\bar{Q}_{\dot{a}}$ can be obtained from $Q_{\dot{a}}$ replacing "bar" variables by non-bar variables and i by $-i$. It can be seen that these superchages annihilate the vacuum (1).

To construct the physical Fock space it is necessary to fix the residual gauge symmetry generated by the world sheet momentum P. This gauge fixing improves the level matching condition on a physical state $|\Phi\rangle$:

$$P|\Phi\rangle = \sum_{n=1}^{\infty} n\left(N_n^B + N_n^F - \bar{N}_n^B - \bar{N}_n^F\right)|\Phi\rangle = 0, \tag{6}$$

where the N^B, N^F, are the bosons and fermions number operators for left-moving modes and the "bar" ones the same for the right-moving modes.

Let us now apply the TFD approach to construct the thermal Fock space for the type IIB superstring. We have first to duplicate the degrees of freedom. To this end we consider a copy of the original Hilbert space, denoted by $\widetilde{H}$. The tilde Hilbert space is built with the set of oscillators operators, $\widetilde{a}_0$, $\widetilde{S}^a$, $\widetilde{a}_n^I$, $\widetilde{\bar{a}}_n^I$, $\widetilde{S}_n^a$, $\widetilde{\bar{S}}_n^a$ that have the same (anti-) commutation properties of the original ones and the operators of the two systems (anti-) commute among themselves.

We can now construct the thermal system. This is achieved by implementing a thermal Bogoliubov transformation in the total (enlarged) Hilbert space. The transformation generator considered in our generalization is given by

$$G = G^B + G^F, \tag{7}$$

for

$$G^B = \sum_{n=1}\left(G_n^B + \bar{G}_n^B\right), \qquad G^F = G_0^F + \sum_{n=1}\left(G_n^F + \bar{G}_n^F\right), \tag{8}$$

where

$$G_0^F = \gamma_{1_0}\widetilde{S}^\dagger \cdot S^\dagger - \gamma_{2_0} S\cdot\widetilde{S} + \gamma_{3_0}\left(S^\dagger\cdot S - \widetilde{S}\cdot\widetilde{S}^\dagger\right), \tag{9}$$

$$G^B = \sum_{n=1}\left[\lambda_{1_n}\widetilde{a}_n^\dagger\cdot a_n^\dagger - \lambda_{2_n} a_n\cdot\widetilde{a}_n + \lambda_{3_n}\left(a_n^\dagger\cdot a_n + \widetilde{a}_n\cdot\widetilde{a}_n^\dagger\right)\right], \tag{10}$$

$$G^F = \sum_{n=1}\left[\gamma_{1_n}\widetilde{S}_n^\dagger\cdot S_n^\dagger - \gamma_{2_n} S_n\cdot\widetilde{S}_n + \gamma_{3_n}\left(S_n^\dagger\cdot S_n - \widetilde{S}_n\cdot\widetilde{S}_n^\dagger\right)\right], \tag{11}$$

with the λ and γ coefficients given by

$$\begin{aligned} \lambda_{1_n} &= \theta^B_{1_n} - i\theta^B_{2_n}, & \lambda_{2_n} &= -\lambda^*_{1_n}, & \lambda_{3_n} &= \theta^B_{3_n}, \\ \gamma_{1_n} &= \theta^F_{1_n} - i\theta^F_{2_n}, & \gamma_{2_n} &= -\gamma^*_{1_n}, & \gamma_{3_n} &= \theta^F_{3_n}, \\ \gamma_{1_0} &= \theta^F_{1_0} - i\theta^F_{2_0}, & \gamma_{2_0} &= -\gamma^*_{1_0}, & \gamma_{3_0} &= \theta^F_{3_0}. \end{aligned} \tag{12}$$

The labels B and F specify fermions and bosons, the dots represent the inner products and θ, $\bar{\theta}$ are real parameters. In the thermal equilibrium they are related to the Bose-Einstein and Fermi-Dirac distributions of the oscillator n as we will see. The expression (10)-(12) are the same for "bar" sector, but replacing operators and parameters by "bar" ones.

The bosonic and fermionic transformed operators can be obtained as follows: Consider an oscillator-like operator, A, that can be commuting or anti-commuting. For this operator one have

$$\begin{pmatrix} A_n^i(\theta) \\ \breve{A}_n^{i\dagger}(\theta) \end{pmatrix} = e^{-iG}\begin{pmatrix} A_n^i \\ \widetilde{A}_n^{i\dagger} \end{pmatrix} e^{iG} = \mathbb{B}_n \begin{pmatrix} A_n^i \\ \widetilde{A}_n^{i\dagger} \end{pmatrix}, \tag{13}$$

$$\begin{pmatrix} A_n^{i\dagger}(\theta) & -\sigma\breve{A}_n^i(\theta) \end{pmatrix} = \begin{pmatrix} A_n^{i\dagger} & -\sigma\widetilde{A}_n^i \end{pmatrix}\mathbb{B}_n^{-1}, \tag{14}$$

where the matrix transformation is given by

$$\mathbb{B}_n = \begin{pmatrix} u_n & v_n \\ \sigma v_n^* & u_n^* \end{pmatrix}, \qquad |u_n|^2 - \sigma|v_n|^2 = 1, \tag{15}$$

with $\sigma = 1$ for bosons and $\sigma = -1$ for fermions. The matrix elements for fermions are

$$u_n \equiv U_n^F = \cosh(i\Gamma_n) + \frac{\gamma_{3_n}}{\Gamma_n}\sinh(i\Gamma_n), \qquad v_n \equiv V_n^F = -\frac{\gamma_{1_n}}{\Gamma_n}\sinh(i\Gamma_n), \tag{16}$$

and Γ_n is defined by the following relation

$$\Gamma_n^2 = -\gamma_{1_n}\gamma_{2_n} + \gamma_{3_n}^2. \tag{17}$$

For bosons one has

$$u_n \equiv U_n^B = \cosh(i\Lambda_n) + \frac{\lambda_{3_n}}{\Lambda_n}\sinh(i\Lambda_n), \qquad v_n \equiv V_n^B = \frac{\lambda_{1_n}}{\Lambda_n}\sinh(i\Lambda_n), \tag{18}$$

and Λ_n is defined by the following relation

$$\Lambda_n^2 = \lambda_{1_n}\lambda_{2_n} + \lambda_{3_n}^2. \tag{19}$$

The expression (13)-(19) for the "bar" sector can be obtained replacing the operators and parameters by "bar" ones. The new conjugation rules in the transformed space, called breve, can be presented as follows: consider any operators $A(\theta)$, $B(\theta)$ in the transformed space. These operators can be fermionic or bosonic. Associated with these operators one has their breve counterparts $\breve{A}(\theta)$, $\breve{B}(\theta)$. The map between both spaces is given by the following breve conjugation rules

$$
\begin{aligned}
\left[A(\theta)\,B(\theta)\right]\breve{} &= \breve{A}(\theta)\,\breve{B}(\theta)\,, && (20)\\
\left[c_1 A(\theta) + c_2 B(\theta)\right]\breve{} &= \left[c_1^* \breve{A}(\theta) + c_2^* \breve{B}(\theta)\right], && (21)\\
\left[A^\dagger(\theta)\right]\breve{} &= \breve{A}^\dagger(\theta)\,, && (22)\\
\left[\breve{A}(\theta)\right]\breve{} &= \sigma A(\theta)\,, && (23)\\
\left[|0(\theta)\rangle\right]\breve{} &= |0(\theta)\rangle\,, && (24)\\
\left[\langle 0(\theta)|\right]\breve{} &= \langle 0(\theta)|\,, && (25)
\end{aligned}
$$

with c_1, $c_2 \in \mathbb{C}$. The expressions (24) and (25) inform that the thermal vacuum obtained from the thermal Bogoliubov transformation with the generator presented at (7) and given by

$$
\begin{aligned}
|0(\theta)\rangle &= e^{-iG}\,|0\rangle\rangle \\
&= \left(U_0^F\right)^8 e^{-\frac{V_0^F}{U_0^F} S_0^\dagger \cdot \widetilde{S}_0^\dagger} \prod_{n=1} \left[\left(\frac{U_n^F}{U_n^B}\right)^8 \left(\frac{\overline{U}_n^F}{U_n^B}\right)^8 e^{-\frac{V_n^B}{U_n^B} a_n^\dagger \cdot \widetilde{a}_n^\dagger - \frac{\overline{V}_n^B}{\overline{U}_n^B} \overline{a}_n^\dagger \cdot \widetilde{\overline{a}}_n^\dagger - \frac{V_n^F}{U_n^F} S_n^\dagger \cdot \widetilde{S}_n^\dagger - \frac{\overline{V}_n^F}{\overline{U}_n^F} \overline{S}_n^\dagger \cdot \widetilde{\overline{S}}_n^\dagger} \right] |0\rangle\rangle, \qquad (26)
\end{aligned}
$$

is invariant under breve conjugation rules. The thermal Fock space is constructed from the above vacuum by applying the thermal creation operators. As the Bogoliubov transformation is canonical, the thermal operators obey the same (anti-) commutators relations as the operators at $T = 0$.

According to the TFD formulation the hamiltonian of the total (duplicated) system in constructed in order to keep untouched the original dynamics of the superstring. This operator can be written as

$$
\widehat{H} = H - \widetilde{H}, \qquad (27)
$$

in such a way that $\widehat{H}$ plays the rôle of the hamiltonian generating the temporal translation in the thermal Fock space. Using the commutation relations we can prove that the Heisenberg equations are satisfied replacing H and $\widetilde{H}$ by $\widehat{H}$.

The TFD approach will be now used to compute thermodynamical quantities by evaluating operators from the original system in the thermal Fock space. It was appointed out by Polchinski [14] that, in the one-string sector, the torus path integral computation of the free energy coincides with what we would obtain by adding the contributions from different states of the spectrum to the free energy. In a recent work [15] it was shown how the closed string thermal torus can be viewed in TFD approach.

The free energy is obtained from the knowledge of the thermal energy and entropy operators. The energy operator is such that the level matching condition must be implemented. In the TFD approach it can be done by considering the shifted hamiltonian:

$$
H_s = H + \frac{2\pi i \lambda}{\beta} P, \qquad (28)
$$

where H, the original hamiltonian, is defined at (4), λ is a lagrange multiplier and P is given at (6). The dependence of the thermal vacuum on the lagrange multiplier comes from the Bogoliubov transformation parameters. This is achieved defining first the free energy like potential

$$\mathcal{F} = \mathcal{E} - \frac{1}{\beta}\mathcal{S}, \tag{29}$$

with

$$\mathcal{E} \equiv \langle 0(\theta)|H_s|0(\theta)\rangle, \qquad \mathcal{S} \equiv \langle 0(\theta)|K|0(\theta)\rangle, \tag{30}$$

where $\mathcal{E}$ is related with thermal energy of the system, and $\mathcal{S}$ with its entropy obtained from the entropy operator, K, defined as follows

$$K = K^B + K^F, \tag{31}$$

where the bosonic entropy operator is given by

$$\begin{aligned} K^B = & -\sum_{n=1}\left\{a_n^\dagger\cdot a_n \ln\left(\frac{\lambda_{1_n}\lambda_{2_n}}{\Lambda_n^2}\sinh^2(i\Lambda_n)\right) - a_n\cdot a_n^\dagger \ln\left(1+\frac{\lambda_{1_n}\lambda_{2_n}}{\Lambda_n^2}\sinh^2(i\Lambda_n)\right)\right\} \\ & -\sum_{n=1}\left\{\bar{a}_n^\dagger\cdot \bar{a}_n \ln\left(\frac{\bar{\lambda}_{1_n}\bar{\lambda}_{2_n}}{\Lambda_n^2}\sinh^2(i\bar{\Lambda}_n)\right) - \bar{a}_n\cdot \bar{a}_n^\dagger \ln\left(1+\frac{\bar{\lambda}_{1_n}\bar{\lambda}_{2_n}}{\Lambda_n^2}\sinh^2(i\bar{\Lambda}_n)\right)\right\}, \end{aligned}$$

and the fermionic one by

$$\begin{aligned} K^F = & -\left\{S_0^\dagger\cdot S_0 \ln\left(\frac{\gamma_{1_0}\gamma_{2_0}}{\Gamma_0^2}\sinh^2(i\Gamma_0)\right) + S_0\cdot S_0^\dagger \ln\left(1-\frac{\gamma_{1_0}\gamma_{2_0}}{\Gamma_0^2}\sinh^2(i\Gamma_0)\right)\right\} \\ & -\sum_{n=1}\left\{S_n^\dagger\cdot S_n \ln\left(\frac{\gamma_{1_n}\gamma_{2_n}}{\Gamma_n^2}\sinh^2(i\Gamma_n)\right) + S_n\cdot S_n^\dagger \ln\left(1-\frac{\gamma_{1_n}\gamma_{2_n}}{\Gamma_n^2}\sinh^2(i\Gamma_n)\right)\right\} \\ & -\sum_{n=1}\left\{\bar{S}_n^\dagger\cdot \bar{S}_n \ln\left(\frac{\bar{\gamma}_{1_n}\bar{\gamma}_{2_n}}{\Gamma_n^2}\sinh^2(i\bar{\Gamma}_n)\right) + \bar{S}_n\cdot \bar{S}_n^\dagger \ln\left(1-\frac{\bar{\gamma}_{1_n}\bar{\gamma}_{2_n}}{\Gamma_n^2}\sinh^2(i\bar{\Gamma}_n)\right)\right\}. \end{aligned}$$

Performing the above expectation values and minimizing the free energy like potential with respect to the parameters it will be a minimum for

$$\begin{aligned} |V_0^F|^2 &= \frac{\gamma_{1_0}\gamma_{2_0}}{\Gamma_0^2}\sinh^2(i\Gamma_0) = \frac{1}{2}, \\ |V_n^F|^2 &= \frac{\gamma_{1_n}\gamma_{2_n}}{\Gamma_n^2}\sinh^2(i\Gamma_n) = \frac{1}{e^{n\left(\frac{\beta}{p^+}+i2\pi\lambda\right)}+1}, \\ |\bar{V}_n^F|^2 &= \frac{\bar{\gamma}_{1_n}\bar{\gamma}_{2_n}}{\Gamma_n^2}\sinh^2(i\bar{\Gamma}_n) = \frac{1}{e^{n\left(\frac{\beta}{p^+}-i2\pi\lambda\right)}+1}, \\ |V_n^B|^2 &= \frac{\lambda_{1_n}\lambda_{2_n}}{\Lambda_n^2}\sinh^2(i\Lambda_n) = \frac{1}{e^{n\left(\frac{\beta}{p^+}+i2\pi\lambda\right)}-1}, \\ |\bar{V}_n^B|^2 &= \frac{\bar{\lambda}_{1_n}\bar{\lambda}_{2_n}}{\Lambda_n^2}\sinh^2(i\bar{\Lambda}_n) = \frac{1}{e^{n\left(\frac{\beta}{p^+}-i2\pi\lambda\right)}-1}. \end{aligned} \tag{32}$$

The above expressions manifest the dependence of the transformation parameters with the lagrange multiplier λ, p^+ and β, and fix the thermal vacuum (26) as those that reproduce the trace over the transverse sector.

Replacing the above results in the free energy like potential (29), one finds, following [9, 10],

$$F = -\frac{l_s}{\beta}\int dp^+ \int_0^1 d\lambda\, e^{\beta p^+} \ln 2^8 e^{-\beta\frac{p^2}{2p^+}} \prod_{n=\mathbb{Z}} \left[\frac{1+e^{-\frac{\beta\omega_n}{p^+}+i\lambda k_n}}{1-e^{-\frac{\beta\omega_n}{p^+}+i\lambda k_n}}\right]^8 . \qquad (33)$$

This expression is the TFD answer for the type IIB Green-Schwarz superstring free energy. The above result is the same of that obtained by using other methods as for example the statistical mechanics operator approach and functional integration, when the world sheet is defined on a torus.

The factor 2^8 comes from the zero modes of the entropy operator. In fact, it is the degeneracy of the ground state and corresponds to the contribution of the 256 massless states of the type IIB superstring to the free energy. With this the TFD generalization presented here seems to be consistent at least for the system studied in this work.

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