PROCEEDINGS

International Europhysics Conference on HEP

CP Violation in a SUSY $SO(10) \times U(2)_F$ Model

K.T. Mahanthappa* **and Mu-Chun Chen**

Department of Physics, University of Colorado, Boulder, CO 80309-0390, U.S.A.
E-mail: ktm@verb.colorado.edu, mu-chun.chen@colorado.edu

ABSTRACT: We construct a model based on SUSY $SO(10)$ combined with $U(2)$ family symmetry including complex phases leading to CP violation. In contrast with the commonly used effective operator approach, $\overline{126}$-dimensional Higgs fields are utilized to construct the Yukawa sector. R-parity symmetry is thus preserved at low energies. The *symmetric* mass textures arising from the left-right symmetry breaking chain of $SO(10)$ give rise to very good predictions for quark and lepton masses and mixings. The prediction for $\sin 2\beta$ agrees with the current bounds from BaBar and Belle. In the neutrino sector, our predictions are in good agreement with results from atmospheric neutrino experiments. Our model favors both the LOW and QVO solutions to the solar neutrino anomaly; the matrix element for neutrinoless double beta decay is highly suppressed. The leptonic analog of the Jarlskog invariant, J^l_{CP}, is predicted to be of $O(10^{-2})$.

$SO(10)$ has long been thought to be an attractive candidate for a grand unified theory (GUT) for a number of reasons: First of all, it unifies all the 15 known fermions with the right-handed neutrino for each family into one 16-dimensional spinor representation. The seesaw mechanism then arises very naturally, and the non-zero neutrino masses can thus be explained. Since a complete quark-lepton symmetry is achieved, it has the promise for explaining the pattern of fermion masses and mixing. Because $B-L$ contained in $SO(10)$ is broken in symmetry breaking chain to the SM, it also has the promise for baryogenesis. Recent atmospheric neutrino oscillation data from Super-Kamiokande indicates non-zero neutrino masses. This in turn gives very strong support to the viability of $SO(10)$ as a GUT group. Models based on $SO(10)$ combined with discrete or continuous family symmetry have been constructed to understand the flavor problem. Most of the models utilize "lopsided" mass textures which usually require more parameters and therefore are less constrained. Furthermore, the right-handed neutrino Majorana mass operators in most of these models are made out of $16_H \times 16_H$ which breaks the R-parity at a very high scale. The aim of this talk, based on Ref.[1, 2], is to present a realistic model based on supersymmetric $SO(10)$ combined with $U(2)$ family symmetry which successfully predicts

*Speaker.

the low energy fermion masses and mixings. Since we utilize *symmetric* mass textures and $\overline{126}$-dimensional Higgs representations for the right-handed neutrino Majorana mass operator, our model is more constrained in addition to having R-parity conserved. We first discuss the viable phenomenology of mass textures followed by the model which accounts for it, and then the implications of the model for neutrino mixing, CP violation, neutrinoless double beta decay and leptogenesis are presented.

The set of up- and down-quark mass matrix combination is given by, at the GUT scale,

$$M_u = \begin{pmatrix} 0 & 0 & ae^{i\gamma_a} \\ 0 & be^{i\gamma_b} & ce^{i\gamma_c} \\ ae^{i\gamma_a} & ce^{i\gamma_c} & e^{i\gamma} \end{pmatrix} dv_u, \qquad M_d = \begin{pmatrix} 0 & ee^{i\gamma_e} & 0 \\ ee^{i\gamma_e} & fe^{i\gamma_f} & 0 \\ 0 & 0 & e^{i\gamma_h} \end{pmatrix} hv_d \tag{1}$$

with $a \simeq b \ll c \ll 1$, and $e \ll f \ll 1$. Symmetric mass textures arise naturally if $SO(10)$ breaks down to the SM through the left-right symmetric breaking chain $SU(4) \times SU(2)_L \times SU(2)_R$. $SO(10)$ relates the up-quark mass matrix to the Dirac neutrino mass matrix, and the down-quark mass matrix to the charged lepton mass matrix. To achieve the Georgi-Jarlskog relations, $m_d \simeq 3m_e$, $m_s \simeq \frac{1}{3}m_\mu$, $m_b \simeq m_\tau$, a factor of -3 is needed in the $(2,2)$ entry of the charged lepton mass matrix,

$$M_e = \begin{pmatrix} 0 & ee^{i\gamma_e} & 0 \\ ee^{i\gamma_e} & -3fe^{i\gamma_f} & 0 \\ 0 & 0 & e^{i\gamma_h} \end{pmatrix} hv_d \tag{2}$$

This factor of -3 can be accounted for by the $SO(10)$ CG coefficients associated with $\overline{126}$-dimensional Higgs representations. In order to explain the smallness of the neutrino masses, we will adopt the type I seesaw mechanism. The Dirac neutrino mass matrix is identical to the mass matrix of the up-quarks in the framework of $SO(10)$

$$M_{\nu_{LR}} = \begin{pmatrix} 0 & 0 & ae^{i\gamma_a} \\ 0 & be^{i\gamma_b} & ce^{i\gamma_c} \\ ae^{i\gamma_a} & ce^{i\gamma_c} & e^{i\gamma} \end{pmatrix} dv_u \tag{3}$$

The right-handed neutrino sector is an unknown sector. It is only constrained by the requirement that it gives rise to a bi-maximal mixing pattern and a hierarchical mass spectrum at low energies. To achieve this, we consider an effective neutrino mass matrix of the form

$$M_{\nu_{LL}} = M_{\nu_{LR}}^T M_{\nu_{RR}}^{-1} M_{\nu_{LR}} = \begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1 \\ t & 1 & 1 \end{pmatrix} \frac{d^2 v_u^2}{M_R} \tag{4}$$

The effective neutrino mass matrix of this form is obtained if the right-handed neutrino mass matrix has the same texture as that of the Dirac neutrino mass matrix,

$$M_{\nu_{RR}} = \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} M_R \tag{5}$$

and if the elements δ_i are of the right orders of magnitudes, determined by $\delta_i = f_i(a, b, c, t, \theta)$, where $\theta \equiv (\gamma_b - 2\gamma_c - \gamma)$. Note that $M_{\nu_{LL}}$ has the same texture as that of $M_{\nu_{LR}}$ and $M_{\nu_{RR}}$. That is to say, the seesaw mechanism is form invariant. A generic feature of mass matrices of the type given in Eq.(4) is that they give rise to bi-maximal mixing pattern. After diagonalizing this mass matrix, one can see immediately that the squared mass difference between $m^2_{\nu_1}$ and $m^2_{\nu_2}$ is of the order of $O(t^3)$, while the squared mass difference between $m^2_{\nu_2}$ and $m^2_{\nu_3}$ is of the order of $O(1)$, in units of Λ. For $t \ll 1$, the phenomenologically favored relation $\Delta m^2_{atm} \gg \Delta m^2_{\odot}$ is thus obtained.

The $U(2)$ family symmetry is implemented *á la* the Froggatt-Nielsen mechanism which simply states that the heaviest matter fields acquire their masses through tree level interactions with the Higgs fields while masses of the lighter matter fields are produced by higher dimensional interactions involving, in addition to the regular Higgs fields, exotic vector-like pairs of matter fields and the so-called flavons (flavor Higgs fields). After integrating out superheavy ($\approx M$) vector-like matter fields, the mass terms of the light matter fields get suppressed by a factor of $\frac{<\theta>}{M}$, where $<\theta>$ is the VEVs of the flavons and M is the UV-cutoff of the effective theory above which the family symmetry is exact. We assume that the family symmetry scale is higher than the GUT scale. The heaviness of the top quark and to suppress the SUSY FCNC together suggest that the third family of matter fields transform as a singlet and the lighter two families of matter fields transform as a doublet under $U(2)$. In the family symmetric limit, only the third family has non-vanishing Yukawa couplings. $U(2)$ breaks down in two steps: $U(2) \stackrel{\epsilon M}{\longrightarrow} U(1) \stackrel{\epsilon' M}{\longrightarrow} nothing$, where $\epsilon' \ll \epsilon \ll 1$ and M is the family symmetry scale. These small parameters ϵ and ϵ' are the ratios of the vacuum expectation values of the flavon fields to the family symmetry scale. Since $\psi_3\psi_3 \sim 1_S$, $\psi_3\psi_a \sim 2$, $\psi_a\psi_b \sim 2 \otimes 2 = 1_A \oplus 3$, the only relevant flavon fields are in the $A^{ab} \sim 1_A$, $\phi^a \sim 2$, and $S^{ab} \sim 3$ dimensional representations of $U(2)$. Because we are confining ourselves to symmetric mass textures, we use only ϕ^a and S^{ab}. In the chosen basis, the VEVs various flavon fields could acquire are given by

$$\frac{\langle\phi\rangle}{M} \sim O\begin{pmatrix} \epsilon' \\ \epsilon \end{pmatrix}, \quad \frac{\langle S^{ab}\rangle}{M} \sim O\begin{pmatrix} \epsilon' & \epsilon' \\ \epsilon' & \epsilon \end{pmatrix} \tag{6}$$

Putting everything together, a symmetric mass matrix would have the following built-in hierarchy given by

$$\begin{pmatrix} \epsilon' & \epsilon' & \epsilon' \\ \epsilon' & \epsilon & \epsilon \\ \epsilon' & \epsilon & 1 \end{pmatrix} \tag{7}$$

Combining $SO(10)$ with $U(2)$, the most general superpotential which respects the symmetry one could write down is given schematically by

$$W = H(\psi_3\psi_3 + \psi_3\frac{\phi^a}{M}\psi_a + \psi_a\frac{S^{ab}}{M}\psi_b) \tag{8}$$

A discrete symmetry $(Z_2)^3$ is needed to avoid unwanted couplings. The field content of our model is then given by

– matter fields

$$\psi_a \sim (16,2)^{-++} \quad (a=1,2), \qquad \psi_3 \sim (16,1)^{+++}$$

– Higgs fields:

$$\begin{array}{lllll} (10,1): & T_1^{+++}, & T_2^{-+-}, & T_3^{--+}, & T_4^{---}, \quad T_5^{+--} \\ (\overline{126},1): & \overline{C}^{---}, & \overline{C}_1^{+++}, & \overline{C}_2^{++-} & \end{array}$$

– Flavon fields:

$$\begin{array}{llll} (1,2): & \phi_{(1)}^{++-}, & \phi_{(2)}^{+-+}, & \Phi^{-+-} \\ (1,3): & S_{(1)}^{+--}, & S_{(2)}^{---}, & \Sigma^{++-} \end{array}$$

and the superpotential of our model which generates fermion masses is given by

$$\begin{array}{l} W = W_{D(irac)} + W_{M(ajorana)} \\ W_D = \psi_3\psi_3 T_1 + \frac{1}{M}\psi_3\psi_a \left(T_2\phi_{(1)} + T_3\phi_{(2)}\right) + \frac{1}{M}\psi_a\psi_b \left(T_4 + \overline{C}\right) S_{(2)} + \frac{1}{M}\psi_a\psi_b T_5 S_{(1)} \\ W_M = \psi_3\psi_3\overline{C}_1 + \frac{1}{M}\psi_3\psi_a\Phi\overline{C}_2 + \frac{1}{M}\psi_a\psi_b\Sigma\overline{C}_2 \end{array} \tag{9}$$

where T_i's and $\overline{C}_i$'s are the 10 and $\overline{126}$ dimensional Higgs representations of $SO(10)$ respectively, and Φ and Σ are the doublet and triplet of $U(2)$, respectively. Detailed quantum number assignment and the VEVs acquired by various scalar fields are given in Ref.[1]. This superpotential gives rise to the mass textures given in Eq.(1)-(5):

$$M_{u,\nu_{LR}} = \begin{pmatrix} 0 & 0 & \langle 10_2^+\rangle \epsilon' \\ 0 & \langle 10_4^+\rangle \epsilon & \langle 10_3^+\rangle \epsilon \\ \langle 10_2^+\rangle \epsilon' & \langle 10_3^+\rangle \epsilon & \langle 10_1^+\rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 & r_2\epsilon' \\ 0 & r_4\epsilon & \epsilon \\ r_2\epsilon' & \epsilon & 1 \end{pmatrix} M_U \tag{10}$$

$$M_{d,e} = \begin{pmatrix} 0 & \langle 10_5^-\rangle \epsilon' & 0 \\ \langle 10_5^-\rangle \epsilon' & (1,-3)\left\langle \overline{126}^- \right\rangle \epsilon & 0 \\ 0 & 0 & \langle 10_1^-\rangle \end{pmatrix} = \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & (1,-3)p\epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} M_D \tag{11}$$

where $M_U \equiv \langle 10_1^+\rangle$, $M_D \equiv \langle 10_1^-\rangle$, $r_2 \equiv \langle 10_2^+\rangle / \langle 10_1^+\rangle$, $r_4 \equiv \langle 10_4^+\rangle / \langle 10_1^+\rangle$ and $p \equiv \left\langle \overline{126}^- \right\rangle / \langle 10_1^-\rangle$. The right-handed neutrino mass matrix is

$$M_{\nu_{RR}} = \begin{pmatrix} 0 & 0 & \left\langle \overline{126}_2^{\prime 0} \right\rangle \delta_1 \\ 0 & \left\langle \overline{126}_2^{\prime 0} \right\rangle \delta_2 & \left\langle \overline{126}_2^{\prime 0} \right\rangle \delta_3 \\ \left\langle \overline{126}_2^{\prime 0} \right\rangle \delta_1 & \left\langle \overline{126}_2^{\prime 0} \right\rangle \delta_3 & \left\langle \overline{126}_1^{\prime 0} \right\rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} M_R \tag{12}$$

with $M_R \equiv \left\langle \overline{126}_1^{\prime 0} \right\rangle$. Note that, since we use $\overline{126}$-dimensional representations of Higgses to generate the heavy Majorana neutrino mass terms, R-parity is preserved at all energies.

With values of $m_f, (f = u, c, t, e, \mu, \tau)$ and those of $|V_{us,ub,cb}|$ at the weak scale, the input parameters at the GUT scale are determined. The predictions for the charged fermion masses and CKM mixing of our model at M_Z which are summarized in Table[1] including 2-loop RGE effects are in good agreements with the experimental values. In the neutrino sector, the LOW solution to the solar neutrino problem is obtained with

	experimental results extrapolated to M_Z	predictions at M_z
$\frac{m_s}{m_d}$	$17 \sim 25$	25
m_s	$93.4^{+11.8}_{-13.0} MeV$	$85.66 MeV$
m_b	$3.00 \pm 0.11 GeV$	$3.147 GeV$
$\lvert V_{ud} \rvert$	$0.9745 - 0.9757$	0.9751
$\lvert V_{cd} \rvert$	$0.218 - 0.224$	0.2218
$\lvert V_{cs} \rvert$	$0.9736 - 0.9750$	0.9744
$\lvert V_{td} \rvert$	$0.004 - 0.014$	0.005358
$\lvert V_{ts} \rvert$	$0.034 - 0.046$	0.03611
$\lvert V_{tb} \rvert$	$0.9989 - 0.9993$	0.9993
J^q_{CP}	$(2.71 \pm 1.12) \times 10^{-5}$	1.748×10^{-5}
$\sin 2\alpha$	$-0.95 \;-\; 0.33$	-0.8913
$\sin 2\beta$	$0.59 \pm 0.14 \pm 0.05$ (BaBar)	0.7416
	$0.58^{+0.32+0.09}_{-0.34-0.10}$ (Belle)	
γ	$34^0 - 82^0$	34.55^0 $(0.6030 rad)$

Table 1: The predictions for the charged fermion masses, the CKM matrix elements and the CP violation measures.

$(\delta_1, \delta_2, \delta_3, M_R) = (0.00125, 2.22 \times 10^{-4} e^{i(0.22)}, 0.0165 e^{-i(0.0017)}, 2.01 \times 10^{13} GeV)$. The atmospheric and solar squared mass differences are predicted to be $\Delta m^2_{23} = 2.95 \times 10^{-3} eV^2$ and $\Delta m^2_{12} = 1.77 \times 10^{-7} eV^2$; the mixing angles are given by $\sin^2 2\theta_{atm} = 0.999$, and $\sin^2 2\theta_{\odot} = 0.994$. $|U_{e\nu_3}|$ is predicted to be 0.0750 which is below the upper bound 0.16 by the CHOOZ experiment. The leptonic Jarlskog invariant is predicted to be $J^l_{CP} = -0.00815$, and the matrix element for the neutrinoless double beta decay is predicted to be $| < m > | = 1.36 \times 10^{-3} eV$. The masses of the three heavy neutrinos are given by $(M_1, M_2, M_3) = (9.41 \times 10^7, 1.49 \times 10^9, 2.01 \times 10^{13}) GeV$. We can also have the QVO solution with $(\delta_1, \delta_2, \delta_3, M_R) = (0.00127, 3.64 \times 10^{-5} e^{i(0.220)}, 0.0150 e^{-i(0.0107)}, 1.22 \times 10^{14} GeV)$. In this case, $\Delta m^2_{23} = 3.12 \times 10^{-3} eV^2$, and $\Delta m^2_{12} = 7.58 \times 10^{-10} eV^2$. The mixing angles are given by $\sin^2 2\theta_{atm} = 0.999$, and $\sin^2 2\theta_{\odot} = 0.995$. $|U_{e\nu_3}|$ is predicted to be 0.0531. J^l_{CP} and $< m >$ are predicted to be -0.00811 and $3.07 \times 10^{-4} eV$ respectively. The masses of the three heavy neutrinos are given by $(M_1, M_2, M_3) = (3.70 \times 10^7, 2.34 \times 10^9, 1.22 \times 10^{14}) GeV$.

A few words concerning baryonic asymmetry are in order. Even though the sphaleron effects destroy baryonic asymmetry, it could be produced as an asymmetry in the generation of $(B - L)$ at a high scale because of lepton number violation due to the decay of heavy right-handed Majorana neutrinos, which in turn is converted into baryonic asymmetry due to sphalerons. But in our model this mechanism produces baryonic asymmetry of $O(10^{-13})$ which is too small to account for the observed value of $(1.7 - 8.3) \times 10^{-11}$, reasons being that the mass of the lightest right-handed Majorana neutrino is too small *and* the $1 - 3$ family mixing of right-handed neutrinos is too large, leading, in essence, to the violation of the out-of-equilibrium condition required by Sakharov. So a mechanism other than leptogenesis is required to explain baryonic asymmetry.

This work was supported, in part, by the U.S. Department of Energy under Grant No. DE FG03-05ER40892.

References

[1] M.-C. Chen and K.T. Mahanthappa, *Phys. Rev.* **D**62, 113007 (2000).

[2] M.-C. Chen and K.T. Mahanthappa, hep-ph/0106093. (This paper contains details and relevant references.)