Supersymmetric Model Contributions to B_d^0–$\bar{B}_d^0$ Mixing and $B \to \pi\pi$, $\rho\gamma$ Decays

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ABSTRACT: By studying the origin of supersymmetry (SUSY) contributions that could impact on B_d^0–$\bar{B}_d^0$ mixing and $B \to \pi\pi$ decay, we find that the former is sensitive to left-left or right-right squark mixings, while the latter is sensitive to left-right squark mixings. These two processes in general are not much correlated in SUSY models. If the smallness of $B \to \pi\pi$ is due to SUSY models, one would likely have large $B \to \rho\gamma$ from chiral enhancement, and the rate could be within present experimental reach. Even if $B \to \rho\gamma$ is not greatly enhanced, it could have large mixing dependent CP violation.

Following earlier measurements on the Golden mode asymmetry $a_{J/\psi K_S}$ the BaBar and Belle Collaborations have recently firmly established [1] $\sin 2\phi_1$ to be nonzero. Combining the two most recent values [1], $0.99 \pm 0.14 \pm 0.06$ (Belle) and $0.59 \pm 0.14 \pm 0.05$ (BaBar), one gets the average value $\sin 2\phi_1 = 0.79 \pm 0.11$. While this is still consistent with the Cabibbo–Kobayashi–Maskawa unitarity (CKM) fit value of $\sin 2\phi_1 = 0.698 \pm 0.066$ [2], the central value is now somewhat on the high side, especially for the Belle number. If this trend persists, it would imply the presence of New Physics.

The first result on the charmless decay mode $B^0 \to \pi^+\pi^-$ was given by the CLEO Collaboration, giving $\mathrm{Br}(B^0 \to \pi^+\pi^-) = (4.3^{+1.6}_{-1.4} \pm 0.5) \times 10^{-6}$ [3]. BaBar and Belle Collaborations also reported recently their results [4], $\mathrm{Br}(B^0 \to \pi^+\pi^-) = (4.1 \pm 1.0 \pm 0.7) \times 10^{-6}$, $(5.9^{+2.4}_{-2.1} \pm 0.5) \times 10^{-6}$, respectively. The combined result with averaged $\mathrm{Br}(B^0 \to \pi^+\pi^-) = 4.4 \pm 0.9$ seem to be on the low side when compared to the SM prediction of $\mathrm{Br}(B^0 \to \pi^+\pi^-) \sim 8 \times 10^{-6}$ [5], for $\phi_3 \sim 60°$. In SM, the tree amplitude dominates over the penguin amplitude, which is about 30% of the former. Thus we may need large contribution from new physics if it is responsible for the smallness of the rate.

As one of the leading candidates for new physics, supersymmetry (SUSY) helps resolve many of the potential problems that emerge when one extends beyond the SM. In the context of SUSY, we then ask the following questions [6] : Is it possible for SUSY models to affect both processes? If so, are they correlated, since both of them are $b \to d$ flavor changing processes? Where can we find other related effects?

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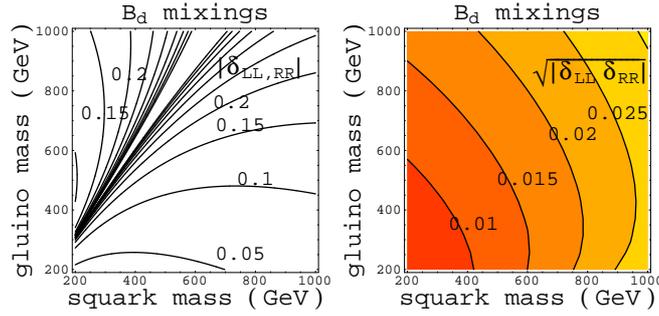
Figure 1: Limits on $\delta_{LL,RR}$ and $\delta_{LL}\delta_{RR}$ obtained by assuming $|M_{12}^{\mathrm{SUSY}}| < |M_{12}^{\mathrm{SM}}|$ in B_d mixing.
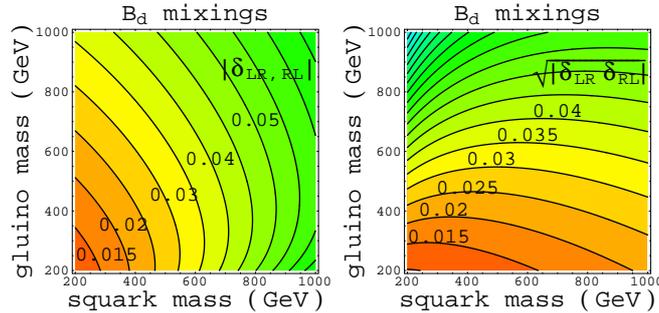
Figure 2: Limits on $\delta_{LR,RL}$ and $\delta_{LR}\delta_{RL}$ obtained by assuming $|M_{12}^{\mathrm{SUSY}}| < |M_{12}^{\mathrm{SM}}|$ in B_d mixing.

We follow the approach of Ref. [7] to analyse SUSY contributions. In quark mass basis, one defines [8], $\delta_{AB} \equiv (\widetilde{m}_d^2)_{AB}^{\widetilde{d}\widetilde{b}}/\widetilde{m}^2$, which is roughly the down squark mixing angle, $(\widetilde{m}_d^2)^{\widetilde{d}\widetilde{b}}$ is the $\widetilde{d}$–$\widetilde{b}$ element of the squark mass matrix, and $A, B = L, R$. Since B_d^0-$\bar{B}_d^0$ mixing is a $\Delta B = 2$ process, we need a power of δ in each of the internal squark lines to change flavor. There are altogether six combinations: δ_{LL}^2, δ_{RR}^2, $\delta_{LL}\delta_{RR}$, δ_{LR}^2, δ_{RL}^2 and $\delta_{LR}\delta_{RL}$.

In Figs. 1 and 2, we show estimated limits of all six $\sqrt{|\delta\,\delta|}$s over the parameter space of sub-TeV gluino and squarks. The limits are taken such that the SUSY contribution to B_d mixing matrix element, $|M_{12}|$, is comparable with the SM result. The limits shown in these figures may serve as upper limits from the Δm_{B_d} constraint on one hand, and serve as roughly the required values to give impact on $a_{J/\psi K_S}$.

We see that the limits on $\sqrt{|\delta_{LL}\delta_{RR}|}$, $|\delta_{LR,RL}|$ and $\sqrt{|\delta_{LR}\delta_{RL}|}$ are all of order few %, with $\sqrt{|\delta_{LL}\delta_{RR}|}$ as the most sensitive source for B_d mixing. However, the limit on $|\delta_{LL,RR}|$ as shown in Fig. 1(a) are of order few 10%. This rather different behavior is due to the possible cancellation between the box and the crossed box diagram contributions, which can weaken the bounds. A total cancellation is reflected in the valley along $x_{\widetilde{g},\widetilde{q}} \equiv m_{\widetilde{g}}^2/\widetilde{m}^2 \sim$ 2.43 where $|\delta_{LL,RR}|$ is not constrained by $|M_{12}^{\mathrm{SUSY}}| \sim |M_{12}^{\mathrm{SM}}|$. The order of magnitude and the behaviors of these figures can be understood [6].

We now turn to the $B \to \pi\pi$ case. Since both generalized factorization and QCD factorization approaches give similar results for this mode [5], we use the former one for simplicity [6]. Gluino contribution is dominant and free from $\mathrm{Br}(B \to X_s\gamma)$ constraint.
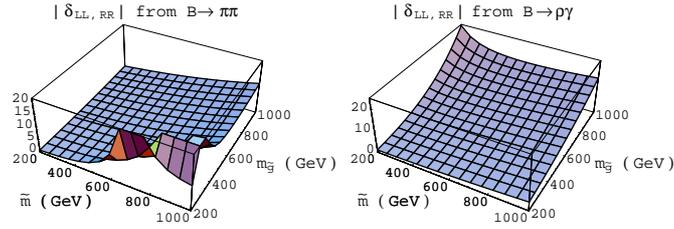
Figure 3: Lower and upper limits on squark mixing angles $\delta_{LL,RR}$ obtained by (a) $\mathrm{Br}^{\mathrm{SUSY}}(\pi^+\pi^-)/\mathrm{Br}^{\mathrm{SM}}(\pi^+\pi^-) > 10\%$, (b) $\mathrm{Br}^{\mathrm{SUSY}}(\rho^0\gamma)/\mathrm{Br}^{\mathrm{SM}}(\rho^0\gamma) < 4$, respectively.
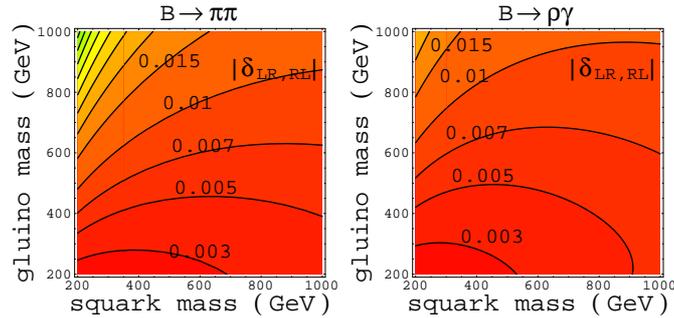
Figure 4: Lower and upper limits on squark mixing angles $\delta_{LR,RL}$ obtained by (a) $\mathrm{Br}^{\mathrm{SUSY}}(\pi^+\pi^-)/\mathrm{Br}^{\mathrm{SM}}(\pi^+\pi^-) > 10\%$, (b) $\mathrm{Br}^{\mathrm{SUSY}}(\rho^0\gamma)/\mathrm{Br}^{\mathrm{SM}}(\rho^0\gamma) < 4$, respectively.

There are two types of diagrams: the gluino box and the gluino penguin. The former as well as the F_1 term (the quark chirality conserving vertex term) of the latter only depend on one power of $\delta_{LL,RR}$. The F_2 term (the quark chirality flipped vertex term) of the gluino penguin contributes through the color dipole term $C_g^{(\prime)}$ with all types of squark mixings. Note that in the large N_c limit, one has the N_c factor for $C_g^{(\prime)}$ only when a gluon attaches to the internal gluino line, which can be easily understood by using the 't Hooft's double line notation. Furthermore, the chiral enhancement factor $m_{\tilde{g}}/m_b$ accompanying $\delta_{LR,RL}$ is a unique feature of the F_2-term.

For a direct destructive interference to cut down by half the predicted SM rate, one needs the SUSY amplitude to be 30% of the SM amplitude. This will be the minimum requirement on the SUSY contribution. In Figs. 3(a) and 4(a) we show limits on $|\delta_{LL,RR}|$ and $|\delta_{LR,RL}|$, respectively. We require the SUSY contribution alone to give 10% of $\mathrm{Br}^{\mathrm{SM}}(B \to \pi\pi)$, corresponding to $\sim 30\%$ in amplitude. From Fig. 3(a), we see that the decay rate is insensitive to left-left and right-right squark mixings, which means gluino box and $\delta_{LL,RR}$ related gluino penguins do not give large contributions.

In Fig. 4(a), we show the required $|\delta_{LR,RL}|$ to produce large enough SUSY contribution in $B \to \pi\pi$ decay. For most of the parameter space a less than 2% mixing angle in left-right mixing is enough to generate such a large SUSY contribution. The sensitivity is greatly enhanced from the previous case due to the chiral enhancement factor $m_{\tilde{g}}/m_b$. Note that there is nothing peculiar about chiral enhancement. It only reflects the chiral suppression
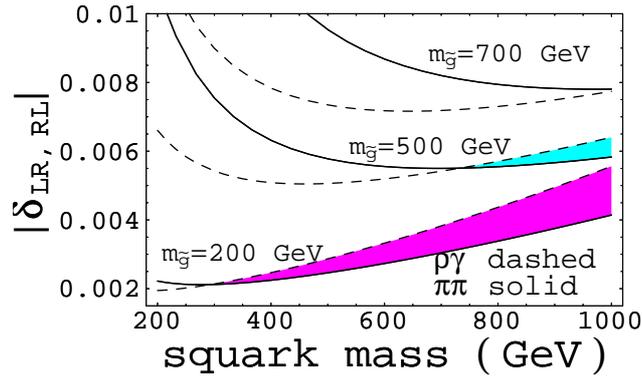
Figure 5: Dashed [solid] lines are upper [lower] bounds on squark mixing angles $\delta_{LR,RL}$ obtained by $\mathrm{Br}^{\mathrm{SUSY}}(\rho^0\gamma)/\mathrm{Br}^{\mathrm{SM}}(\rho^0\gamma) < 4$ $[\mathrm{Br}^{\mathrm{SUSY}}(\pi^+\pi^-)/Br^{\mathrm{SM}}(\pi^+\pi^-) > 10\%]$ with $m_{\tilde{g}} = 200, 500, 700$ GeV, respectively. Shaded regions are allowed parameter space.

of C_g in the SM due to the V−A nature of the weak interaction, which need not be obeyed by interactions beyond the SM. The behavior of Fig 4(a) can be understood [6].

Recently, the Belle Collaboration reports a 90% upper limit on $\mathrm{Br}(B^0 \to \rho^0\gamma) < 1.06 \times 10^{-5}$ [9], nominally ~ 5 times the SM prediction. We require the decay rate due to the SUSY contribution alone to be smaller than 4 times the SM prediction. In Fig. 3(b) and 4(b), we show the limits on $|\delta_{LL,RR}|$ and $|\delta_{LR,RL}|$, respectively. Similar to the $B \to \pi\pi$ case, the decay rate is insensitive to $|\delta_{LL,RR}|$, but very sensitive to $|\delta_{LR,RL}|$, as expected. We note that in most of the parameter space shown in Fig. 4(b), $|\delta_{LR,RL}|$ are constrained to be less than 2%. When compared to Fig. 4(a), in most of the parameter space $|\delta_{LR,RL}|$ impacts more on $B \to \rho\gamma$ than $B \to \pi\pi$. As a pure loop process the former is more sensitive to new physics. As shown in Fig. 5, it is quite interesting that in the parameter space of $\tilde{m} \sim 300$–1000 GeV and $m_{\tilde{g}} \leq 700$ GeV, with mixing angle $\sim 0.2\%$–0.8%, the model gives sizable contribution to $B \to \pi\pi$ decay that can account for the smallness of the rate, but still satisfy the $B \to \rho\gamma$ constraint. This is due to an extra $\ln x_{\tilde{g}\tilde{q}}$ enhancement factor in the loop function of $C_g^{(\prime)}$ such that gluino penguin can give larger contribution in $b \to dg$ than in $b \to d\gamma$ process [6].

For illustration, in Fig. 6 we show $\mathrm{Br}(\pi^+\pi^-)$ obtained by using $m_{\tilde{g}} = 200$ GeV, $\tilde{m} =$
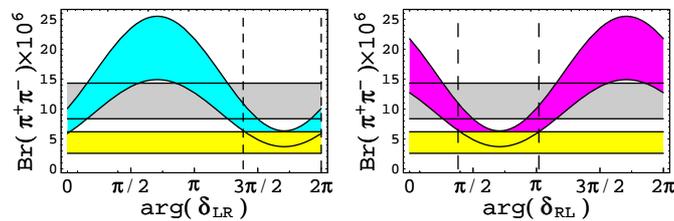
Figure 6: $\mathrm{Br}(B \to \pi\pi)$ obtained by using $\tilde{m} = 800$ GeV, $m_{\tilde{g}} = 200$ GeV, and (a) $|\delta_{LR}| = 0.0035$, (b) $|\delta_{RL}| = 0.0035$, respectively. The upper band corresponds to the SM prediction, while the lower band corresponds to the experimental result with 2σ error range.

800 GeV, and (a) $|\delta_{LR}| = 0.0035$, $\delta_{RL} = \delta_{LL,RR} = 0$, (b) $|\delta_{RL}| = 0.0035$, $\delta_{LR} = \delta_{LL,RR} = 0$, respectively. The upper (lower) band corresponds to the SM prediction (the averaged experimental result $\text{Br}(\pi^+\pi^-) = 4.4 \pm 0.9$ with 2σ error range). With $\arg(\delta_{LR,RL})$ within the dashed lines, i.e. $\arg(\delta_{LR}) \sim 4.3$–$2\pi$, $\arg(\delta_{RL}) \sim 1.2$–$3.2$, $\text{Br}(\pi^+\pi^-)$ can be brought down by SUSY contributions to the experimental range. On the other hand, the strength factor of $a_{\rho\gamma}$, $\sin 2\theta \equiv 2|C_\gamma C'_\gamma|/(|C_\gamma|^2 + |C'_\gamma|^2)$ [10], can be as large as 90% in this case. The measurability of the asymmetry in $B \to \rho\gamma$ decay is better than in $B \to K^*\gamma$ [10].

Left-left and/or right-right $\tilde{d}$–$\tilde{b}$ mixings with few % to few 10% mixing angle can generate large enough contribution to B_d-mixing that could deviate $a_{J/\psi K_S}$ from its SM value. As shown in Fig. 3(b), such squark mixing angles are safe from the $B \to \rho\gamma$ constraint. However, as shown in Fig. 3(a), one needs large mixing with a sizable mass splitting to affect the $B \to \pi\pi$ decay rate in this case. Such a large mixing is already ruled out by the experimental measurement of Δm_{B_d}, as shown in Figs. 1(a) and 1(b), unless one fine tunes the parameter space to be very close to $x_{\tilde{g}\tilde{q}} = 2.43$, and turn off left-left or right-right mixings. It is much easier to compete with the SM box diagram and modify $\sin 2\phi_1$ than to compete with tree dominated $B \to \pi\pi$ decay.

Alternatively, left-right and/or right-left $\tilde{d}$–$\tilde{b}$ mixings with few % mixing angles could also give sizable contribution to B_d mixing. However, due to the amplification effect of the chiral enhancement, the size of this mixing angle is severely constrained by $B \to \rho\gamma$ to be less than 2% in most of the parameter space given in Fig. 4(b). It cannot give sizable contribution to B_d mixing, as one can tell by comparing Figs. 2 and 4(b). It is interesting that there is parameter space where $m_{\tilde{g}}$ is suitably light and the mixing angle $\delta_{LR,RL}$ is less than 1%, where the model gives sizable contribution to $B \to \pi\pi$ decay without violating the $B \to \rho\gamma$ constraint. In other words, we need left-right and/or right-left mixings rather than left-left and/or right-right mixings to affect $B \to \pi\pi$ decay. Thus, if the smallness of $\text{Br}(B \to \pi\pi)$ is due to SUSY, it is likely that one will have large effects in $b \to d\gamma$, including rate enhancement and mixing induced asymmetry.

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