

# T and CPT in B-Factories

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ABSTRACT: For the  $B_d$  meson system, CP, T and CPT indirect violation can be described using two physical parameters,  $\varepsilon$  and  $\delta$ . The traditional observables based on flavour tag and used in the kaon system, are not helpful in the  $B_d$  case, and new asymmetries have to be introduced. Here such alternative observables, based on CP tag, are presented, together with the first estimation on the sensitivity that current asymmetric B-factories can achieve on their measurement.

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## 1. Introduction

Violation of CP, T and CPT symmetries in the time evolution of  $K^0-\bar{K}^0$  was studied by the CP-LEAR experiment [1] from the preparation of definite flavour states. The study of this *flavour-to-flavour* evolution allows the construction of observables which violate CP and T, or CP and CPT. In order to be non-vanishing, nevertheless, these observables need the presence of an absorptive part in the effective Hamiltonian that governs neutral meson system. The different lifetimes of physical states  $K_L$  and  $K_S$  provides this ingredient. In the case of  $B_d$  mesons, on the contrary, the width difference  $\Delta\Gamma$  between the physical states is expected to be negligible[2], so that the T- and CPT-odd observables proposed for kaons, and based on flavour tag, will practically vanish for a  $B_d$  system.

Here alternative observables are discussed, which allow the study of CP, T and CPT indirect violation in the  $B_d$  system[3]. Based on CP-tag[4], these observables do not need the presence of  $\Delta\Gamma \neq 0$ , and can be constructed from the entangled states of  $B_d$  mesons.

In the following section, the invariant parameters  $\varepsilon$  and  $\delta$  are introduced to describe indirect violation of symmetries in the neutral meson system. In section 3 we describe the CP tag of  $B_d$  from the entangled states in a B-factory. Next, section 4 reviews three different kinds of asymmetries that can be constructed from these states, namely, *flavour-to-flavour* and both genuine and non-genuine *CP-to-flavour* asymmetries. Finally, in section 5 the first estimates on the reach and sensitivity of the experimental analysis are given.

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## 2. Invariant description of CP, T and CPT violation in the $B$ system

The physical states in the neutral  $B$ -meson system are a linear combination of the definite flavour  $B^0$  and  $\bar{B}^0$ . Physical states can also be written in terms of CP eigenstates,  $|B_{\pm}\rangle \equiv \frac{1}{\sqrt{2}}(I \pm CP)|B^0\rangle$ , which are physical iff the CP operator is well defined. To do so, one has to introduce two complex parameters,  $\varepsilon_{1,2}$ , to describe the CP mixing, so that  $|B_{1(2)}\rangle = \frac{1}{\sqrt{1+|\varepsilon_{1(2)}|^2}} [|B_{+(-)}\rangle + \varepsilon_{1(2)}|B_{- (+)}\rangle]$ . The complex parameters  $\varepsilon_{1,2}$ , invariant under rephasing of the meson states, are better interpreted in terms of  $\varepsilon \equiv (\varepsilon_1 + \varepsilon_2)/2$  and  $\delta \equiv \varepsilon_1 - \varepsilon_2$ , whose observable character is explicit when they are written in terms of the effective hamiltonian matrix elements [5].

Discrete symmetries impose different restrictions on the effective mass matrix,  $H = M - \frac{i}{2}\Gamma$ , and thus on the invariant parameters  $\varepsilon$  and  $\delta$ :

- CPT invariance requires<sup>1</sup>  $H_{11} = H_{22}$ , so that  $\delta = 0$ , with no restriction on  $\varepsilon$ ;
- T invariance imposes  $\text{Im}(M_{12}\text{CP}_{12}^*) = \text{Im}(\Gamma_{12}\text{CP}_{12}^*) = 0$ , and so  $\varepsilon = 0$ ;
- and CP conservation requires both  $\varepsilon = \delta = 0$ .

In the exact limit  $\Delta\Gamma = 0$ , an approximation that is expected to be excellent for the  $B_d$  system, both  $\text{Re}(\varepsilon)$  and  $\text{Im}(\delta)$  vanish. Then  $\text{Im}(\varepsilon) \neq 0$  is a proof of both CP and T violation, and  $\text{Re}(\delta) \neq 0$  is a proof of CP and CPT violation, but neither  $\text{Re}(\varepsilon) = 0$  nor  $\text{Im}(\delta) = 0$  are proof of a fundamental invariance. Information on the symmetry parameters can be extracted from the study of time evolution of  $B$  meson entangled states.

## 3. CP-Tag from entangled states

In a  $B$  factory operating at the  $\Upsilon(4S)$  peak, correlated pairs of neutral  $B$ -mesons are produced through  $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ . Charge conjugation together with Bose statistics require the initial state to be  $|i\rangle = \frac{1}{\sqrt{2}}(|B^0(\vec{k}), \bar{B}^0(-\vec{k})\rangle - |\bar{B}^0(\vec{k}), B^0(-\vec{k})\rangle)$ . This permits the performance of a flavour tag: if at  $t_0$  one of the mesons decays through a channel  $X$ , which is only allowed for one flavour, the other meson in the pair must have the opposite flavour at  $t_0$ , and will later evolve during  $\Delta t = t - t_0$  until its final decay to some state  $Y$ .

The entangled  $B - \bar{B}$  state can also be expressed in terms of the CP eigenstates as  $|i\rangle = \frac{1}{\sqrt{2}}(|B_-, B_+\rangle - |B_+, B_-\rangle)$ . Thus it is also possible to carry out a CP tag, if we have a CP-conserving decay into a definite CP final state  $X$ , so that its detection allows us to identify the decaying meson as a  $B_+$  or a  $B_-$ , which decays into  $Y$  after a time  $\Delta t$ . In Ref. [4] we described how this determination is possible and unambiguous to  $\mathcal{O}(\lambda^3)$ , the flavour-mixing parameter of the CKM matrix.

If we consider only decay channels  $X, Y$  which are either flavour or CP conserving, then the final configuration  $(X, Y)$  corresponds to a single particle mesonic transition. The intensity for the final configuration,  $I(X, Y, \Delta t) \equiv \frac{1}{2} \int_{\Delta t}^{\infty} dt' |(X, Y)|^2$  is proportional to the time dependent probability for the meson transition.

<sup>1</sup>Here  $H_{ij}$ ,  $M_{ij}$ , and so on, represent the matrix elements in the flavour basis  $B^0 - \bar{B}^0$ .

## 4. Asymmetries

By comparing the probabilities corresponding to different processes we build time-dependent asymmetries that can be classified into three types.

### 4.1 Flavour-to-flavour genuine asymmetries

The final configuration denoted by  $(\ell, \ell)$ , with flavour definite (for example, semileptonic) decays detected on both sides of the detector, corresponds to *flavour-to-flavour* transition at the meson level. The equivalence is shown in Table 1.

The first two processes in the Table are conjugated under CP and also under T. The corresponding Kabir asymmetry[6] is, to linear order in the CPT violating  $\delta$ ,

$$A(\ell^+, \ell^+) \approx \frac{4 \frac{\text{Re}(\varepsilon)}{1+|\varepsilon|^2}}{1+4 \frac{\text{Re}(\varepsilon)}{1+|\varepsilon|^2}}, \quad (4.1)$$

which does not depend on time. However, in the exact limit  $\Delta\Gamma = 0$ ,  $\text{Re}(\varepsilon)$  vanishes, and this quantity will be zero. For the  $B_d$  system, experimental limits on  $\text{Re}(\varepsilon)$  are of few parts in a thousand[7][8].

A second asymmetry arises from the last two processes in Table 1, related by a CP or a CPT transformation,

$$A(\ell^+, \ell^-) \approx -2 \frac{\text{Re}\left(\frac{\delta}{1-\varepsilon^2}\right) \sinh \frac{\Delta\Gamma\Delta t}{2} - \text{Im}\left(\frac{\delta}{1-\varepsilon^2}\right) \sin(\Delta m\Delta t)}{\cosh \frac{\Delta\Gamma\Delta t}{2} + \cos(\Delta m\Delta t)}, \quad (4.2)$$

which is an odd function of time. This asymmetry also vanishes unless  $\Delta\Gamma \neq 0$ . Present limits [7] on  $\text{Im}(\delta)$  are at the level of few percent.

### 4.2 CP-to-flavour genuine asymmetries

Alternative asymmetries can be constructed making use of the CP eigenstates, which can be identified in this system by means of a CP tag. If the first decay product,  $X$ , is a CP eigenstate produced along the CP-conserving direction, i.e. the decay is free of CP

violation, and  $Y$  is a flavour definite channel, then the mesonic transition corresponding to the configuration  $(X, Y)$  is of the type *CP-to-flavour*.

In Table 2 we show the mesonic transitions, with their related final configurations, connected by genuine symmetry transformations to  $B_+ \rightarrow B^0$ .

Comparing the intensities of the four processes, we may construct three genuine asymmetries, namely  $A(\text{CP})$ ,  $A(\text{T})$  and  $A(\text{CPT})$  [3].

$$A(\text{CP}) = -2 \frac{\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \sin(\Delta m\Delta t) + \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{2\text{Re}(\delta)}{1+|\varepsilon|^2} \sin^2\left(\frac{\Delta m\Delta t}{2}\right), \quad (4.3)$$

$(X, Y)$	Meson Transition
$(\ell^+, \ell^+)$	$\bar{B}^0 \rightarrow B^0$
$(\ell^-, \ell^-)$	$B^0 \rightarrow \bar{B}^0$
$(\ell^+, \ell^-)$	$\bar{B}^0 \rightarrow \bar{B}^0$
$(\ell^-, \ell^+)$	$B^0 \rightarrow B^0$

**Table 1:** *Flavour-to-flavour* transitions.

$(X, Y)$	Transition	Transformation
$(J/\Psi K_S, \ell^-)$	$B_+ \rightarrow \bar{B}^0$	CP
$(\ell^-, J/\Psi K_L)$	$B^0 \rightarrow B_+$	T
$(\ell^+, J/\Psi K_L)$	$\bar{B}^0 \rightarrow B_+$	CPT

**Table 2:** Transitions connected to  $(J/\Psi K_S, \ell^+)$ .

the CP odd asymmetry, contains both T-violating and CPT-violating contributions, which are, respectively, odd and even functions of  $\Delta t$ . This asymmetry corresponds to the "gold plate" decay [9] and has been measured recently [10]. T and CPT violating terms can be separated by constructing other asymmetries.

$$A(\text{T}) = -2 \frac{\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \sin(\Delta m \Delta t) \left[ 1 - \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{2\text{Re}(\delta)}{1+|\varepsilon|^2} \sin^2 \left( \frac{\Delta m \Delta t}{2} \right) \right], \quad (4.4)$$

the T asymmetry, needs  $\varepsilon \neq 0$ , and turns out to be purely odd in  $\Delta t$  in the limit we are considering.

$$A(\text{CPT}) = \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} \frac{2\text{Re}(\delta)}{1+|\varepsilon|^2} \frac{\sin^2 \left( \frac{\Delta m \Delta t}{2} \right)}{1 - 2 \frac{\text{Im}(\varepsilon)}{1+|\varepsilon|^2} \sin(\Delta m \Delta t)}, \quad (4.5)$$

is the CPT asymmetry. It needs  $\delta \neq 0$ , and includes both even and odd time dependences.

The above expressions correspond to the limit  $\Delta\Gamma = 0$ , but, being genuine observables, a possible absorptive part could not induce by itself a non-vanishing asymmetry.

### 4.3 CP-to-flavour non-genuine asymmetries

The construction of the quantities described in the previous paragraphs requires to tag both  $B_+$  and  $B_-$  states, and thus the reconstruction of the experimentally challenging decay  $B \rightarrow J/\Psi K_L$ . Conversely, non-genuine asymmetries offer a possibility

$(X, Y)$	Transition	Transformation
$(J/\Psi K_S, \ell^-)$	$B_+ \rightarrow B^0$	CP
$(\ell^+, J/\Psi K_S)$	$\bar{B}^0 \rightarrow B_-$	$\Delta t$
$(\ell^-, J/\Psi K_S)$	$\bar{B}^0 \rightarrow B_-$	$\Delta t + \text{CP}$

**Table 3:** Final configurations with only  $J/\Psi K_S$ .

to measure the symmetry parameters from the reconstruction of  $J/\Psi K_S$  only. But they involve the discrete transformation that we denote  $\Delta t$ , consisting of the exchange in the order of appearance of decay products  $X$  and  $Y$ , which cannot be associated with any fundamental symmetry.

Table 3 shows the different transitions we may study from such final states. Besides the genuine CP asymmetry, there are two new quantities that can be constructed from the comparison between  $(J/\Psi K_S, \ell^+)$  and the processes in the table. In the exact limit  $\Delta\Gamma = 0$ ,  $\Delta t$  and T operations are found to become equivalent, so that the temporal asymmetry satisfies  $A(\Delta t) \equiv A(\ell^+, J/\Psi K_S) = A(\text{T})$  and moreover  $A(\text{CP}\Delta t) \equiv A(\ell^-, J/\Psi K_S) = A(\text{CPT})$ . Since this result holds for  $\Delta\Gamma \approx 0$ , it is expected to be valid for the  $B_d$  system, but not for  $B_s$  and even less for  $K$ . The asymmetries  $A(\Delta t)$  and  $A(\text{CP}\Delta t)$  are non-genuine, and the presence of  $\Delta\Gamma \neq 0$  may induce non-vanishing values for them, even if there is no true T or CPT violation. These fake effects, nevertheless, can be calculated and are thus controllable.

## 5. CP, T, CPT indirect violation reach at asymmetric B-Factories

The asymmetries described in the previous section can be already constructed from the current data taken at Asymmetric B-Factories [10]. The experimental analysis is based on a simultaneous unbinned likelihood fit of the flavour and CP intensities  $I(X, Y; \Delta t)$ ,

together with the  $B^0/\bar{B}^0$  mistag rates and the  $\Delta t$  resolution function. The coefficients of terms with different temporal dependencies contain the information on the symmetry parameters.

From a simulation study, estimations on the reachable statistical precision for the relevant parameters have been calculated for  $\approx 60 \text{ fb}^{-1}$  (assuming yields from Ref. [10]) and are shown in Table 4.

Parameter	(Generated)	Statistical error
$\frac{\text{Re}(\delta)}{1+ \varepsilon ^2}$	(0)	0.09
$\frac{\text{Re}(\varepsilon)}{1+ \varepsilon ^2}$	(0)	0.007
$\frac{\Delta\Gamma}{\Gamma}$	(0)	0.07
$\frac{\text{Im}(\varepsilon)}{1+ \varepsilon ^2}$	(0.35)	0.04
$\Delta m$	(0.472 ps $^{-1}$ )	0.009

## 6. Conclusions

**Table 4:** Projections for  $60 \text{ fb}^{-1}$ .

We have shown how the two complex rephasing invariant parameters  $\varepsilon$  and  $\delta$  describe CP, T and CPT indirect violation in  $B^0 - \bar{B}^0$ . In the exact limit  $\Delta\Gamma = 0$  the number of parameters is reduced to  $\text{Im}(\varepsilon)$  and  $\text{Re}(\delta)$ . Observables based on *flavour-to-flavour* transitions are sensitive to  $\text{Re}(\varepsilon)$ , but need  $\Delta\Gamma \neq 0$ , and thus are not promising in B-factories. Conversely, these experimental facilities allow the construction of new asymmetries based on combination of flavour and CP tags.

First estimations on the sensitivity reachable on B-factories have been presented. This data will be crucial to achieve the separation of the two ingredients: on one hand CP and T violation, described by  $\varepsilon$ , and on the other CP and CPT violation, given by  $\delta$ .

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