EW corrections to $e^+e^- \to \gamma Z$, ZZ at LEP2 in an effective field theory framework

Daniel Gómez Dumm*

Departamento de Física, Comisión Nacional de Energía Atómica,
Av. Libertador 8250, (1429) Buenos Aires, Argentina
E-mail: dumm@venus.fisica.unlp.edu.ar

Abstract: We propose an effective field theory approach to estimate the size of the electroweak corrections to the $e^+e^- \to \gamma Z$ and $e^+e^- \to ZZ$ cross sections at LEP2. Our predictions are shown to agree within 1% with one-loop Standard Model calculations.

1. Introduction

The knowledge of radiative corrections up to definite order for different processes is necessary to perform accurate tests of the Standard Model (SM), allowing to probe the quantum structure of the theory and to search for possible effects of new physics. In most cases, however, the required theoretical analysis reaches an extremely complicated level. In this work [1] we show how the effective field theory (EFT) [2, 3, 4] can help in the estimate of the electroweak (EW) corrections at LEP2 energies, using the precise measurements of LEP1 and SLD.

The standard approach to EW radiative corrections in the SM requires firstly the evaluation of those corrections for LEP1/SLD observables, in order to extract the relevant SM parameters from the available experimental data. It is seen that the extracted values depend strongly on the top–quark mass, m_t, and (to a lesser extent) on the Higgs mass, m_H. Then one can calculate, in terms of these parameters, the radiative corrections to LEP2 observables. As expected, the results also show a strong dependence on m_t and m_H. However, the latter is cancelled almost completely by the m_t and m_H dependences of the input parameters extracted from LEP1/SLD. This is not surprising, since for both LEP1 and LEP2 energies top quarks and Higgs bosons are always virtual. It is then conceivable that a description in terms of an effective theory without explicit top quarks or Higgs bosons is good enough for both LEP1 and LEP2. All top quark and Higgs–boson mass dependences will be absorbed in the effective Lagrangian parameters, which can be determined at LEP1/SLD and then used to make predictions for LEP2 that will be trivially independent on m_t and m_H. We will show that this program can be carried out basically at tree level, achieving precisions for LEP2 predictions at the % level, which should be enough for most purposes.

We will focus on the neutral gauge boson production at LEP2, $e^+e^- \to \gamma Z$ and $e^+e^- \to ZZ$. For these processes, the corrections can be split in two, $\delta^{\rm QED}$ and $\delta^{\rm EW}$, where the former refers to the "pure" QED —or photonic— contributions, while the latter stands for the remaining, non–QED EW piece. We will concentrate here on estimating $\delta^{\rm EW}$, which in the SM involves huge formulae even at one loop [5, 6], showing that within the framework of our EFT the analysis can be carried out with good accuracy in a very simple way.

This note is organized as follows: in Section 2 we discuss how to obtain an effective Lagrangian in the large m_t limit. Then, in Section 3, this Lagrangian is used to calculate the dominant radiative corrections for $e^+e^- \to ZZ$

*Work done in collaboration with A. A. Akhundov, J. Bernabéu and A. Santamaria, University of Valencia, Spain.

and $e^+e^- \to \gamma Z$. In Section 4 we assume that our effective Lagrangian is valid for both LEP1 and LEP2 energies, and use experimental LEP1 and SLD results to fit the Lagrangian parameters. In Section 5 we use the effective theory to give predictions for $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$ cross sections at LEP2. Finally in Section 6 we present our conclusions.

2. Effective EW Lagrangian for LEP energies

An effective Lagrangian for $\mu \leq m_t$ can be obtained from the Standard theory by integrating the top quark at $\mu = m_t$. This can be done at the level of one loop, computing all diagrams containing at least one top quark [3]. The procedure implies a redefinition of the gauge boson fields, which leads to a redefinition of the corresponding couplings. As a consequence, the initially unique coupling constant g splits into g_+ and g_3 below the top–quark mass scale, whereas the Higgs vacuum expectation value v splits into $v_+(\mu) \equiv v + \delta v_+(\mu)$ and $v_3(\mu) \equiv v + \delta v_3(\mu)$.

To get the effective Lagrangian at LEP scales $\mu \simeq m_Z$ it is necessary to perform the matching of the effective theory to the full theory at the scale $\mu = m_t$, and then to scale down using the renormalization group equations for each of the parameters $g_+(\mu)$, $\delta v_+(\mu)$, etc. In addition, one has to diagonalize the neutral gauge boson sector, including a further wave function renormalization of the Z field to absorb a crossed term which originates in the mixing between the W_3 and B wave functions. One ends up with the effective Lagrangian [1, 4]

$$\begin{aligned}\mathcal{L}_{\rm eff} &= W^+_\mu \partial^2 W^{-\mu} + m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} A_\mu \partial^2 A^\mu \\ &+ \frac{1}{2} Z_\mu \partial^2 Z^\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + i\bar{b}\partial\!\!\!/ b \\ &+ \bar\psi\, iD\!\!\!\!/\,\Big[\frac{e_W(m_Z)}{s_Z} W^+, \frac{e_Z(m_Z)}{s_Z c_Z} Z, e(m_Z) A\Big]\,\psi \\ &- \frac{e_Z(m_Z)}{2 s_Z c_Z} \bar{b}\gamma_\mu \left(g_V^b - g_A^b \gamma_5\right) b Z^\mu \\ &+ \frac{1}{3} e(m_Z) \bar{b}\gamma_\mu b A^\mu \ , \end{aligned} \tag{2.1}$$

where c_Z (s_Z) stands for the cosine (sine) of an effective weak mixing angle θ_W at the scale m_Z. Owing to the rescaling needed by the Z field and the splitting of the Higgs vacuum expectation value, the Lagrangian (2.1) has been written in terms of effective couplings $e_Z(m_Z)$ and $e_W(m_Z)$, which at the one loop level turn out to be shifted from the electromagnetic coupling $e(m_Z)$. It is also natural to define

$$\alpha(m_Z) \equiv \frac{e^2(m_Z)}{4\pi} \equiv \frac{\alpha}{1-\Delta\alpha} \ , \tag{2.2}$$

$$\alpha_{Z,W}(m_Z) \equiv \frac{e^2_{Z,W}(m_Z)}{4\pi} \equiv \alpha(m_Z)\,(1+\delta\alpha_{Z,W}) \, , \tag{2.3}$$

where $\alpha \simeq 1/137$ is the fine structure constant and $\Delta\alpha$ is the QED shift produced by the running from its on–shell value to $\mu = m_Z$. In the large m_t limit, the additional shifts $\delta\alpha_{Z,W}$ can be calculated to be [1]

$$\begin{aligned}\delta\alpha_Z &\simeq \frac{\alpha}{12\pi s_Z^2 c_Z^2} \log\left(\frac{m_t}{m_Z}\right) \ , \\ \delta\alpha_W &\simeq \frac{\alpha}{12\pi s_Z^2} \log\left(\frac{m_t}{m_Z}\right) \ .\end{aligned} \tag{2.4}$$

On the other hand, the physical W and Z masses can be trivially related to the parameters $e^2_{W,Z}(m_Z)$, $v_{+,3}(m_Z)$ and s_Z^2. Thus, one can obtain the sine of the Sirlin weak mixing angle, $s_W = (1 - m_W^2/m_Z^2)^{1/2}$, in terms of the sine of the effective mixing angle at the scale m_Z. If we write this relation as

$$s_Z^2 = s_W^2 + \delta s_W^2 \, , \tag{2.5}$$

we get [1]

$$\delta s_W^2 = \frac{\alpha}{\pi}\left[\frac{3}{16 s_Z^2}\frac{m_t^2}{m_Z^2} + \frac{(3-2s_Z^2)}{12 s_Z^2}\log\left(\frac{m_t}{m_Z}\right)\right] . \tag{2.6}$$

It is worth noticing that the coupling of the bottom quark to the Z boson gets special contributions due to the vertex one–loop diagrams involving a virtual top quark. These contributions can be taken into account by parameterizing the effective couplings g_V^b and g_A^b in terms of a parameter $\epsilon_b(m_Z)$ (for details see [4]).

3. $e^+e^- \to \gamma Z$ and $e^+e^- \to ZZ$ in the large m_t limit

Now we can use the Lagrangian (2.1) at tree level, together with the results in the previous section, to estimate the dominant electroweak corrections to $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$ at LEP2 in the large m_t limit. Our calculations are performed in the $\overline{\rm MS}$ scheme, choosing a renormalization scale $\mu = m_Z$. In general, if this is the only scale involved in the processes, the one–loop corrections are expected to be suppressed by a factor $\alpha/\pi \sim 1/500$.

By computing the diagrams in Fig. 1 we easily obtain the cross section for $e^+e^- \to ZZ$. It is the usual tree–level result obtained in the SM, but expressed in terms of the effective couplings $\alpha_Z(m_Z)$ and s_Z^2:

$$\left(\frac{d\sigma^{ZZ}}{d\Omega}\right)_{\rm eff} = \frac{\alpha_Z^2(m_Z)}{32 s_Z^4 c_Z^4}\,\frac{\beta}{s}\,(g_V^4 + 6 g_V^2 g_A^2 + g_A^4) \times \left[\frac{s^2+6m_Z^4}{ut} - \frac{m_Z^4(s-2m_Z^2)^2}{(ut)^2} - 2\right], \quad (3.1)$$

where s, t, u are the usual Mandelstam variables, while $g_V = -1/2 + 2s_Z^2$, $g_A = -1/2$ and $\beta = (1-4m_Z^2/s)^{1/2}$.

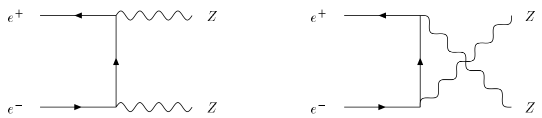

Figure 1: Tree–level contributions to $e^+e^- \to ZZ$.

The accuracy of this effective cross section can be tested by comparing with explicit one–loop calculations in the SM, which can be found if Ref. [6]. There, the Born cross section is defined in terms of the fine structure constant α and the Sirlin weak mixing angle s_W, that is, our expression (3.1), but changing $\alpha_Z(m_Z) \to \alpha$ and $s_Z \to s_W$. In our framework, the leading electroweak corrections in the large m_t limit can be easily obtained using relations (2.3) and (2.5):

$$\delta^{{\rm EW}\,(ZZ)} = 2\,\Delta\alpha + 2\,\delta\alpha_Z + \frac{s_Z^4 c_Z^4}{(g_V^4 + 6g_V^2 g_A^2 + g_A^4)} \times \frac{d}{ds_Z^2}\left[\frac{(g_V^4 + 6g_V^2 g_A^2 + g_A^4)}{s_Z^4 c_Z^4}\right]\delta s_W^2\,, \quad (3.2)$$

with $\delta\alpha_Z$ and δs_W^2 given by (2.4) and (2.6) respectively. We have checked explicitly the resulting expression against the one obtained in [6] by taking there the large m_t limit, and found complete agreement.

The situation is similar in the case of γZ production. As in the previous case, we begin by writing the SM lowest–order differential cross section for the process, which is obtained from the diagrams in Fig. 1 after replacing one of the Z bosons by a photon. Here we find

$$\left(\frac{d\sigma^{\gamma Z}}{d\Omega}\right)_{\rm eff} = \frac{\alpha\,\alpha_Z(m_Z)}{4 s_Z^2 c_Z^2}\,(g_V^2 + g_A^2) \times \frac{(s-m_Z^2)}{s^2}\left(\frac{s^2+m_Z^4}{2ut} - 1\right). \quad (3.3)$$

However, there is now a crucial point when the photon is real, that is with $q^2 = 0$. In that case, choosing a renormalization scale $\mu = m_Z$, one finds that the photon self–energy diagrams of Fig. 2 contain large logarithms, which effectively produce the "running back" of the electromagnetic coupling constant from $\alpha(m_Z)$ to $\alpha(m_e) \simeq \alpha$. Thus, the use of our effective Lagrangian at the scale $\mu = m_Z$ has to be supplemented with the rule that a real photon couples with its on–shell coupling.

The electroweak corrections are now given by

$$\delta^{{\rm EW}\,(\gamma Z)} = \Delta\alpha + \delta\alpha_Z + \frac{s_Z^2 c_Z^2}{(g_V^2 + g_A^2)} \times \frac{d}{ds_Z^2}\left[\frac{(g_V^2 + g_A^2)}{s_Z^2 c_Z^2}\right]\delta s_W^2\,. \quad (3.4)$$

Once again, we have confirmed this result by comparing with analytical calculations in the SM (in this case we have considered one–loop results for the crossed reaction $e^-\gamma \to e^- Z$ [7]).

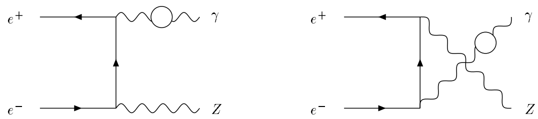

Figure 2: Photon self–energy diagrams contributing to $e^+e^- \to \gamma Z$.

4. Global fit for LEP1/SLD observables

The analytical results of the previous section, obtained in the large m_t limit, still do not account for important corrections such as Higgs–mass and finite top–mass effects. As an alternative procedure, we can use our effective Lagrangian at tree level with arbitrary couplings, and fit those couplings with LEP1/SLD observables. In this way, the effective couplings will contain not only the leading top–quark and Higgs mass dependences but also other universal non–leading corrections.

We have chosen for our fit a set of twelve LEP1 and SLD observables [1]. From the Lagrangian (2.1), it is immediate to see that, at the lowest order, the corresponding analytical expresions in the EFT scheme are basically the same as in the SM, just taking $e_Z(m_Z)/(s_Z c_Z)$ and s_Z as the weak $Z\bar{f}f$ coupling constant and the sine of the Weinberg angle respectively. Only special care has to be taken in the case of the $Z\bar{b}b$ coupling, which requires the additional inclusion of the above mentioned parameter $\epsilon_b(m_Z)$. The parameters to be fitted are five: m_Z, $\alpha_Z(m_Z)$, s_Z^2, $\epsilon_b(m_Z)$ and $\alpha_s(m_Z)$.

The result of our fit is $m_Z = 91.1867 \pm 0.0020$, $\alpha_Z(m_Z) = 0.007788 \pm 0.000012$, $s_Z^2 = 0.23103 \pm 0.00021$, $\alpha_s(m_Z) = 0.1215 \pm 0.0052$ and $\epsilon_b(m_Z) = -0.0053 \pm 0.0023$, with $\chi^2/\mathrm{ndf} = 2.6/7$. It can be seen that all our predictions deviate less than 1.5σ from the measured values (the full table of results can be found in Ref. [1]). This is reflected in the very low value for χ^2, and shows that for LEP1 and SLD data the EFT approach works remarkably well.

5. Predictions for $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$ at LEP2

Once the effective couplings at the scale of m_Z have been determined, we can proceed to evaluate the magnitude of the electroweak corrections for processes to be measured at LEP2. Notice that the running of the parameters from $\mu = m_Z$ to the relevant LEP2 scale of 190 GeV should be small, and in principle can be neglected. In addition, for the processes we are considering here, namely $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$, the gauge bosons are on–shell, therefore the relevant scale is fixed by their masses.

Let us take the EFT tree–level expressions (3.1) and (3.3) for $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$ respectively, with α_Z, m_Z and s_Z^2 taken from the fit, and compute the size of the deviations from the Born cross sections expressed in the on–shell scheme. For the process $e^+e^- \to ZZ$ we obtain

$$\delta_{\mathrm{eff}}^{\mathrm{EW}\,(ZZ)} \simeq 5.4 \pm 0.4\ \%\,. \tag{5.1}$$

This value can be contrasted with the result of full one–loop calculations in the SM. We consider the analytical expressions in Ref. [6], using the on–shell value for s_W^2, together with $m_t = 168 \pm 8$ GeV (arising from Z–pole analysis [8]) and $m_H \approx m_Z$. This gives

$$\delta_{\mathrm{SM-1\ loop}}^{\mathrm{EW}\,(ZZ)} \simeq 5.3 \pm 1.0\ \%\,, \tag{5.2}$$

where the error is mainly due to the uncertainty in m_t. As can be seen, the agreement between the values in (5.1) and (5.2) is remarkably good. In addition, it can be seen [6] that the size of the corrections is almost independent of the scattering angle θ. This is also consistent with our approach, since the shifts of α and s_W^2 from the on–shell to the effective values lead only to a global correction.

It is important to remark that the value in (5.1) has been found in a quite straightforward way, whereas that in (5.2) can be obtained only after a lengthy analytical calculation of the full one–loop corrections for the process in the SM. In addition, the one–loop result, though in principle more precise, depends on various uncertain quantities, such as the top and Higgs masses and the running of the electromagnetic coupling.

A similar procedure can be carried out for the case of $e^+e^- \to \gamma Z$, taking into account that the value of the electromagnetic coupling to be used in (3.3) is the on–shell fine structure constant α. We get

$$\delta_{\mathrm{eff}}^{\mathrm{EW}\,(\gamma Z)} \simeq 3.7 \pm 0.2\ \%\,, \tag{5.3}$$

which can be compared with known calculations [7] for the crossed reaction $e^-\gamma \to e^-Z$ (which shows exactly the same dependence on the parameters α_Z and s_Z). For a center–of–mass energy of 100 GeV and a top mas of 168 GeV we find

$$\delta_{\mathrm{SM-1\ loop}}^{\mathrm{EW}\,(e^-\gamma\to e^-Z)} \simeq 3.1 \pm 0.4\%\,. \tag{5.4}$$

That means, our result (5.3) lies within the expected level of accuracy. Once again, it can be seen that δ^{EW} is rather independent of the scattering angle.

Finally, notice that up to now we have compared the size of the EW corrections with reference to Born cross sections. In order to get a better estimate of the accuracy of our approach, we can instead compare directly the values for the cross sections arising from both the EFT and SM one–loop analyses. In this way the comparison is much less sensitive to the top–quark mass, which does not appear explicitly in $\sigma_{\rm eff}$. We compute the ratio

$$\Delta \equiv \frac{\sigma_{\rm eff} - \sigma_{\rm SM-1\ loop}}{\sigma_{\rm SM-1\ loop}} \ , \tag{5.5}$$

obtaining

$$\Delta^{(ZZ)} = 0.0012 \pm 0.0038 \ , \tag{5.6}$$

$$\Delta^{(\gamma Z)} = 0.0065 \pm 0.0017 \ . \tag{5.7}$$

It is seen that in both cases the agreement between EFT and one–loop SM values is better than 1%.

6. Conclusions

In this note we show how the effective field theory can be used to estimate the size of electroweak corrections for double gauge boson production at LEP2 energies.

We consider here an effective EW Lagrangian which arises when the top quark is integrated out, and use this Lagrangian at tree level to obtain analytical formulae, in the large m_t limit, for the differential cross sections for $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$ at LEP2. The results agree completely with full one–loop EW calculations. Then, to go beyond the large m_t limit, we use an effective Lagrangian similar to that arising from the EFT, but leaving the couplings as free parameters, and we fit these parameters from present LEP1 and SLD data. The results of this fit (five parameters, twelve LEP1/SLD observables) are amazingly good: in all cases the difference between fitted and experimental values is less than 1.5σ. Finally, with the effective couplings taken from the fit, we compute the differential cross sections for $e^+e^- \to ZZ$ and $e^+e^- \to \gamma Z$ at LEP2 energies, using the effective Lagrangian at tree level. The results are compared with the values obtained using full one–loop calculations in the SM, and for both processes the agreement is found to be better than 1 %. Our hope is now that the effective Lagrangian approach could be extended to estimate the size of the EW corrections for other important LEP2 processes, which in some cases are rather hard to evaluate using standard calculations.

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