$O(\alpha_s)$ Corrections to Longitudinal Spin-Spin Correlations in $e^+e^- \to q\bar{q}$

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ABSTRACT: We calculate the $O(\alpha_s)$ corrections to longitudinal spin-spin correlations in $e^+e^- \to q\bar{q}$. For top quark pair production the $O(\alpha_s)$ corrections to the longitudinal spin-spin asymmetry amount to less than 1% in the q^2-range from above $t\bar{t}$-threshold up to $\sqrt{q^2} = 1000\,GeV$. In the $e^+e^- \to b\bar{b}$ case the $O(\alpha_s)$ corrections reduce the asymmetry value from its $m = 0$ value of -1 to approximately -0.96 for q^2-values around the Z-peak.

Recently there has been renewed interest in the role of quark mass effects in the production of quarks and gluons in e^+e^- annihilations. Jet definition schemes, event shape variables, heavy flavour momentum correlations and the polarization of the gluon [1] are affected by the presence of quark masses for charm and botton quarks even when they are produced at the scale of the Z-mass [2, 3, 4]. A careful investigation of quark mass effects in e^+e^- annihilations may even lead to a alternative determination of the quark mass values [2, 3, 4, 5]. There is obvious interest in quark mass effects for $t\bar{t}$ production where quark mass effects cannot be neglected in the envisaged range of energies to be covered by the Next Linear Collider (NLC).

We present the $O(\alpha_s)$ radiative corrections to longitudinal spin-spin correlations of massive quark pairs produced in e^+e^- annihilations. The longitudinal polarization of massive quarks affects the shape of the energy spectrum of their secondary decay leptons. Thus longitudinal spin-spin correlation effects in pair produced quarks and antiquarks will lead to correlation effects of the energy spectra of their secondary decay leptons and antileptons. Let us begin with defining the differential joint quark-antiquark density matrix $d\boldsymbol{\sigma} = d\sigma_{\lambda_1\lambda_2;\lambda_1'\lambda_2'}$ where λ_1 and λ_2 denote the helicities of the quark and antiquark, respectively. In this paper our main interest is the longitudinal polarization of the quark and antiquark and in particular their longitudinal spin-spin correlations. Thus we specify to the diagonal case $\lambda_1 = \lambda_1'$ and $\lambda_2 = \lambda_2'$.

The diagonal part of the differential joint density matrix can be represented in terms of its components along the products of the unit matrix and the z-components of the Pauli matrix $\boldsymbol{\sigma}_3$ ($\boldsymbol{\sigma}_3 = \hat{p_1}\vec{\boldsymbol{\sigma}}$ for the quark and $\boldsymbol{\sigma}_3 = \hat{p_2}\vec{\boldsymbol{\sigma}}$ for the antiquark, $\hat{p_i} = \vec{p_i}/|\vec{p_i}|$). One has

$$\boldsymbol{\sigma} = \frac{1}{4}\Big(d\sigma \mathbb{1}\otimes\mathbb{1} + d\sigma^{(\ell_1)}\boldsymbol{\sigma}_3\otimes\mathbb{1} + d\sigma^{(\ell_2)}\mathbb{1}\otimes\boldsymbol{\sigma}_3 + d\sigma^{(\ell_1\ell_2)}\boldsymbol{\sigma}_3\otimes\boldsymbol{\sigma}_3\Big). \quad (1)$$

From CP invariance one knows that $d\sigma$ and $d\sigma^{(\ell_1\ell_2)}$ obtain contributions from the parity-even VV- and AA-current products, whereas $d\sigma^{(\ell_1)}$ and $d\sigma^{(\ell_2)}$ are contributed to by the parity-odd VA and AV-current products. The parity-even terms $d\sigma$ and $d\sigma^{(\ell_1\ell_2)}$ are C-even and thus symmetric under $q \leftrightarrow \bar{q}$ exchange, whereas the parity-odd terms are C-odd and thus one has $d\sigma^{(\ell_1)}(p_1, p_2) = -d\sigma^{(\ell_2)}(p_2,p_1)$ for the single-spin dependent contributions.

$O(\alpha_s)$ radiative corrections to the rate com-

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ponent $d\sigma$ have been discussed before [6, 7] including beam polarization effects [8] and beam-event correlation effects [8, 9]. As concerns the longitudinal spin-spin correlation component $d\sigma^{(\ell_1\ell_2)}$ the $O(\alpha_s)$ tree graph contributions have been determined in [10]. Here we calculate the $O(\alpha_s)$ radiative corrections to the fully integrated spin-spin correlation component $\sigma^{(\ell_1\ell_2)}$ where we average out beam-event correlation effects.

As before we write the electro-weak cross section $e^+e^- \to q(p_1)\bar{q}(p_2)$ and $e^+e^- \to q(p_1)\bar{q}(p_2)$ $g(p_3)$ in modular form in terms of two building blocks [8]. Thus we write (beam polarization effects not included and beam-event correlations averaged out)

$$\begin{aligned} d\sigma(s_1^\ell, s_2^\ell) = \frac{1}{4}\Big(& g_{11}(d\sigma_1 + d\sigma_1^{(\ell_1\ell_2)} s_1^\ell s_2^\ell) \\ & + g_{12}(d\sigma_2 + d\sigma_2^{(\ell_1\ell_2)} s_1^\ell s_2^\ell) \\ & + g_{14}(d\sigma_4^{(\ell_1)} s_1^\ell + d\sigma_4^{(\ell_2)} s_2^\ell)\Big). \end{aligned} \tag{2}$$

The index $i = 1, 2, 4$ on the rate components is explained later on. The first building block g_{ij} $(i, j = 1, 2)$ specifies the electro-weak model dependence of the e^+e^- cross section. For the present discussion we need the components g_{11}, g_{12} and g_{14}. They are given by

$$\begin{aligned} g_{11} &= Q_f^2 - 2Q_f v_e v_f Re\,\chi_Z + (v_f^2 + a_f^2)\Xi, \\ g_{12} &= Q_f^2 - 2Q_f v_e v_f Re\,\chi_Z + (v_f^2 - a_f^2)\Xi, \\ g_{14} &= 2Q_f v_e a_f Re\,\chi_Z - 2v_f a_f \Xi, \\ \text{with} \quad & \Xi = (v_e^2 + a_e^2)|\chi_Z|^2 \end{aligned} \tag{3}$$

where, in the Standard Model, $\chi_Z(q^2) = gM_Z^2 q^2/(q^2 - M_Z^2 + iM_Z\Gamma_Z)$, with M_Z and Γ_Z the mass and width of the Z^0 and $g = G_F(8\sqrt{2}\pi\alpha)^{-1} \approx 4.49 \cdot 10^{-5}\,GeV^{-2}$. Q_f are the charges of the final state quarks to which the electro-weak currents directly couple; v_e and a_e, v_f and a_f are the electro-weak vector and axial vector coupling constants. For example, in the Weinberg-Salam model, one has $v_e = -1 + 4\sin^2\theta_W$, $a_e = -1$ for leptons, $v_f = 1 - \frac{8}{3}\sin^2\theta_W$, $a_f = 1$ for up-type quarks ($Q_f = \frac{2}{3}$), and $v_f = -1 + \frac{4}{3}\sin^2\theta_W$, $a_f = -1$ for down-type quarks ($Q_f = -\frac{1}{3}$). In this paper we use Standard Model couplings with $\sin^2\theta_W = 0.226$ and $M_Z = 91.178\,GeV$, $\Gamma_Z = 2.487\,GeV$.

The second building block is determined by the hadron dynamics, i.e. by the current-induced production of a quark-antiquark pair which, in the $O(\alpha_s)$ case, is followed by gluon emission. In the $O(\alpha_s)$ case one also has to add the one loop-contribution. We shall work in terms of unpolarized and polarized hadron tensor components H_{U+L}, $H_{U+L}^{(\ell_1)}$, $H_{U+L}^{(\ell_2)}$ and $H_{U+L}^{(\ell_1\ell_2)}$. In the two body case $e^+e^- \to q\bar{q}$ the unpolarized rate components are given by

$$\sigma^i = \frac{\pi\alpha^2 v}{3q^4} H_{U+L}^i. \tag{4}$$

In the three-body case $e^+e^- \to q\bar{q}g$ the unpolarized differential rate components and the unpolarized hadron tensor components H_{U+L}^i are related by

$$\frac{d\sigma^i}{dydz} = \frac{\alpha}{48\pi q^2} H_{U+L}^i(y, z) \quad (i = 1, 2). \tag{5}$$

As kinematic variables we use the two energy-type variables $y = 1 - 2p_1 q/q^2$ and $z = 1 - 2p_2 q/q^2$.

The index $i = 1, 2$ in Eqs. (4) and (5) specifies the current composition in terms of the two parity-even products of the vector and the axial vector currents according to (dropping all further indices on the hadron tensor)

$$H_{\mu\nu}^1 = \frac{1}{2}(H_{\mu\nu}^{VV} + H_{\mu\nu}^{AA}) \quad H_{\mu\nu}^2 = \frac{1}{2}(H_{\mu\nu}^{VV} - H_{\mu\nu}^{AA}). \tag{6}$$

The notation closely follows the one in [8]. Thus the nomenclature $(U + L)$ in Eqs. (4) and (5) denotes the total rate (U: unpolarized transverse, L: longitudinal) after averaging over the relative beam-event orientation.

Let us begin with listing the Born term contributions to the various polarized and unpolarized two-body hadron tensor components. One has ($\xi = 4m_q^2/q^2$, $v = \sqrt{1-\xi}$)

$$\begin{aligned} H_{U+L}^1(Born) &= (4 - \xi)N_C q^2, \\ H_{U+L}^2(Born) &= 3\xi N_C q^2, \\ H_{U+L}^{1(\ell_1\ell_2)}(Born) &= -(4 - 3\xi)N_C q^2, \\ H_{U+L}^{2(\ell_1\ell_2)}(Born) &= -\xi N_C q^2, \\ H_{U+L}^{4(\ell_1)}(Born) &= 4vN_C q^2, \\ H_{U+L}^{4(\ell_2)}(Born) &= -4vN_C q^2. \end{aligned} \tag{7}$$

The $O(\alpha_s)$ spin dependent hadronic three-body tensor

$$H_{\mu\nu}(p_1,p_2,p_3,s_1,s_2) = \sum_{\text{G spin}} \langle q\bar{q}g|j_\mu|0\rangle\langle 0|j_\mu^\dagger|q\bar{q}g\rangle \tag{8}$$

can easily be calculated from the relevant Feynman diagrams. The $(U+L)$-component is then obtained by contraction with the four-transverse metric tensor $(-g_{\mu\nu}+q_\mu q_\nu/q^2)$. Finally, the longitudinal spin components of the quark and antiquark can be projected out with the help of the respective longitudinal spin vectors. They read

$$(s_1^\ell)^\mu = \frac{s_1^\ell}{\sqrt{\xi}}(\sqrt{(1-y)^2-\xi};0,0,1-y) \tag{9}$$

$$(s_2^\ell)^\mu = \frac{s_2^\ell}{\sqrt{\xi}}(\sqrt{(1-z)^2-\xi};(1-z)\sin\theta_{12},0,$$
$$(1-z)\cos\theta_{12}) \tag{10}$$

with

$$\cos\theta_{12} = \frac{yz+y+z-1+\xi}{\sqrt{(1-y)^2-\xi}\sqrt{(1-z)^2-\xi}}. \tag{11}$$

The resulting spin-independent and single-spin dependent components of the hadron tensor have been given before (see e.g. [6, 8, 7]). Here we list the spin-spin dependent piece. One has ($v_y := \sqrt{(1-y)^2-\xi}$, $v_z := \sqrt{(1-z)^2-\xi}$)

$$H^{1(\ell_1\ell_2)}_{U+L}(y,z) = \frac{1}{v_y v_z}\Bigg[-4(12-10\xi+\xi^2)$$
$$+(1-\xi)(4-3\xi)\xi\left(\frac{1}{y^2}+\frac{1}{z^2}\right)+(4-3\xi)\times$$
$$(8-7\xi)\left(\frac{1}{y}+\frac{1}{z}\right)+2(12-5\xi)(y+z)$$
$$-2(4-\xi)(y^2+z^2)-(4-3\xi)\xi\Bigg(\frac{y}{z^2}-\frac{y^2}{z^2}+$$
$$+\frac{z}{y^2}-\frac{z^2}{y^2}\Bigg)-2(1-\xi)(2-\xi)(4-3\xi)\frac{1}{yz}$$
$$-(4-\xi)(6-5\xi)\times\left(\frac{y}{z}+\frac{z}{y}\right)$$
$$+2(4-5\xi)\left(\frac{y^2}{z}+\frac{z^2}{y}\right)+4\xi yz\Bigg], \tag{12}$$

$$H^{2(\ell_1\ell_2)}_{U+L}(y,z) = \frac{\xi}{v_y v_z}\Bigg[-4\xi+(1-\xi)\xi\left(\frac{1}{y^2}+\frac{1}{z^2}\right)$$
$$+(8-7\xi)\left(\frac{1}{y}+\frac{1}{z}\right)-6(y+z)-2(y^2+z^2)$$
$$-\xi\left(\frac{y}{z^2}-\frac{y^2}{z^2}+\frac{z}{y^2}-\frac{z^2}{y^2}\right)-2(2-\xi)(1-\xi)\frac{1}{yz}$$
$$-(6+\xi)\left(\frac{y}{z}+\frac{z}{y}\right)+2\left(\frac{y^2}{z}+\frac{z^2}{y}\right)-4yz\Bigg]. \tag{13}$$

What remains to be done is to perform the phase space integrations and to add in the one-loop contributions. In this calculation we perform the requisite two-fold phase space integration over the full (y,z) phase space. As in [6, 8] the infrared singularities are regularized by introducing a gluon mass. The infrared singularities in the tree graph and one-loop contributions cancel and one remains with finite remainders. It is quite clear that the finite result is independent of the specific regularization procedure.

For the sake of completeness we include in our results also the unpolarized hadron tensor components which are needed for the normalization of the longitudinal spin-spin asymmetry. The $O(\alpha_s)$ corrections (tree plus loop) read

$$H^1_{U+L}(\alpha_s) = N\Bigg[\frac{3}{2}(4-\xi)(2-\xi)v+\frac{1}{4}(192+$$
$$-104\xi-4\xi^2+3\xi^3)t_3-2(4-\xi)\Big((2-\xi)\times$$
$$(t_8-t_9)+2v(t_{10}+2t_{12})\Big)\Bigg], \tag{14}$$

$$H^2_{U+L}(\alpha_s) = N\xi\Bigg[\frac{3}{2}(18-\xi)v+\frac{3}{4}(24-8\xi-\xi^2)\times$$
$$\times t_3-6\Big((2-\xi)(t_8-t_9)+2v(t_{10}+2t_{12})\Big)\Bigg], \tag{15}$$

$$H^{1(\ell_1\ell_2)}_{U+L}(\alpha_s) = N\Bigg[(88-78\xi-5\xi^2+3\xi^3)\frac{1}{2v}+$$
$$-(2-\xi)(20+3\xi)+\sqrt{\xi}(32+12\xi+3\xi^2)$$
$$-\Big(16-42\xi+31\xi^2-4\xi^3+8(4-3\xi)v^3\Big)\frac{t_3}{v^2}$$
$$+2(4-3\xi)\Big((2-\xi)(t_8-t_{16})+2v(t_{10}+2t_{12})+$$
$$-(4-\xi)(8-3\xi-\xi^2)\frac{t_{13}}{4v^2}\Big)-2(8-10\xi+$$
$$+\xi^2)t_{14}+(32-88\xi+76\xi^2-19\xi^3)\frac{t_{15}}{v^3}\Bigg], \tag{16}$$

$$H^{2(\ell_1\ell_2)}_{U+L}(\alpha_s) = N\xi\Bigg[-(54-65\xi+3\xi^2)\frac{1}{2v}$$

$$+58 - 56\sqrt{\xi} - 3\xi - 3\xi\sqrt{\xi} + \Bigg(2 + \xi - 4\xi^2 + \\ -8v^3\Bigg)\frac{t_3}{v^2} + 2\Big((2-\xi)(t_8 - t_{16}) + 2v(t_{10} + 2t_{12})\Big) \\ +(96 - 140\xi + 35\xi^2 - 3\xi^3)\frac{t_{13}}{2v^2} - 2(10 + \\ +3\xi)t_{14} + (8 - 20\xi + 13\xi^2)\frac{t_{15}}{v^3}\Bigg] \qquad (17)$$

where we have used an overall normalization factor $N = \alpha_s N_C C_F q^2/4\pi v$. The unpolarized hadron tensor components $H^1_{U+L}(\alpha_s)$ and $H^2_{U+L}(\alpha_s)$ including the $O(\alpha_s)$ rate functions t_i ($i = 3, 8, 9, 10, 12$) have been calculated before in [8]. In addition to the rate functions calculated in [8] the spin-spin contributions bring in a set of new rate functions t_i ($i = 13, 14, 15, 16$). The new set of rate functions needed in the present application is given by

$$t_{13} = \ln\left(\frac{1+v}{2-\sqrt{\xi}}\right)$$

$$t_{14} = \ln\left(\frac{4}{\xi}\right)\ln\left(\frac{1+v}{2-\sqrt{\xi}}\right) + 2\mathrm{Li}_2\left(\frac{2-\sqrt{\xi}}{2}\right) + \\ -2\mathrm{Li}_2\left(\frac{\sqrt{\xi}}{2}\right) + \mathrm{Li}_2\left(\frac{1-v}{2}\right) - \mathrm{Li}_2\left(\frac{1+v}{2}\right),$$

$$t_{15} = \left(\ln\left(\frac{1+v}{1-v}\right) + \ln\left(\frac{\sqrt{\xi}}{2-\sqrt{\xi}}\right)\right)^2 \\ -4\mathrm{Li}_2\left(\sqrt{\frac{1-v}{1+v}}\right) + 2\mathrm{Li}_2\left(\frac{2-\sqrt{\xi}}{1+v}\right) + \\ +2\mathrm{Li}_2\left(\frac{1-v}{2-\sqrt{\xi}}\right), \qquad (18)$$

$$t_{16} = \ln\left(\frac{1+v}{1-v}\right)\ln\left(\frac{4v^4}{\xi(1+v)^2}\right) \\ -\mathrm{Li}_2\left(\frac{2v}{(1+v)^2}\right) + \mathrm{Li}_2\left(\frac{-2v}{(1-v)^2}\right) \\ +\frac{1}{2}\mathrm{Li}_2\left(-\frac{(1-v)^2}{(1+v)^2}\right) - \frac{1}{2}\mathrm{Li}_2\left(-\frac{(1+v)^2}{(1-v)^2}\right).$$

We are now in the position to discuss the normalized longitudinal spin-spin correlation function $\langle P^{\ell\ell}\rangle$ which is defined as

$$\langle P^{\ell\ell}\rangle = \frac{\sigma^{(\ell_1\ell_2)}}{\sigma} = \frac{\sigma(\uparrow\uparrow) - \sigma(\uparrow\downarrow) - \sigma(\downarrow\uparrow) + \sigma(\downarrow\downarrow)}{\sigma(\uparrow\uparrow) + \sigma(\uparrow\downarrow) + \sigma(\downarrow\uparrow) + \sigma(\downarrow\downarrow)}. \qquad (19)$$

The mean $\langle P^{\ell\ell}\rangle$ is taken with regard to all phase-space variables including the beam-event orientation variables.

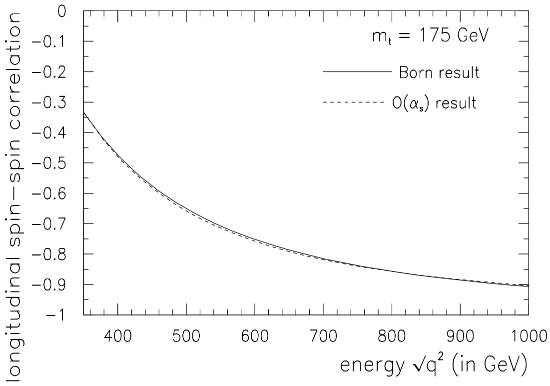

Figure 1: Energy dependence of $O(1)$ and $O(\alpha_s)$ mean longitudinal spin-spin correlations $\langle P^{\ell\ell}\rangle$ in $e^+e^- \to t\bar{t}(g)$

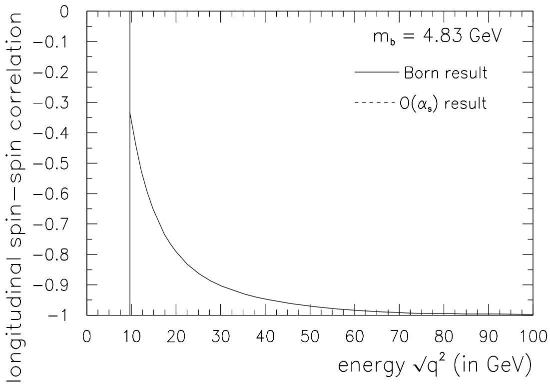

Figure 2: Energy dependence of $O(1)$ and $O(\alpha_s)$ mean longitudinal spin-spin correlations $\langle P^{\ell\ell}\rangle$ in $e^+e^- \to b\bar{b}(g)$. The vertical line indicates the $b\bar{b}$-threshold

Let us now present our numerical results. In Fig. 1 we plot the mean longitudinal spin-spin asymmetry $\langle P^{\ell\ell}\rangle$ against the c.m. energy $\sqrt{q^2}$ for top quark pair production. The longitudinal spin-spin asymmetry rises from its threshold value of $\langle P^{\ell\ell}\rangle = -1/3$ to around $\langle P^{\ell\ell}\rangle = -0.9$ at $\sqrt{q^2} = 1000\,GeV$. The $O(\alpha_s)$ correction to the asymmetry amounts to less than 1% in the q^2-range from above $t\bar{t}$ threshold to $\sqrt{q^2} = 1000\,GeV$. In Fig. 2 we present our results on $\langle P^{\ell\ell}\rangle$ for bottom quark pair production starting from $b\bar{b}$ threshold (where $\langle P^{\ell\ell}\rangle = -1/3$) up to $\sqrt{q^2} = 100\,GeV$. For the lower q^2-values from threshold to about $30\,GeV$ the $O(\alpha_s)$ corrections are quite small. Starting at around $\sqrt{q^2} = 30\,GeV$ the $O(\alpha_s)$ correction become larger. The Born term contribution very quickly acquires its asymp-

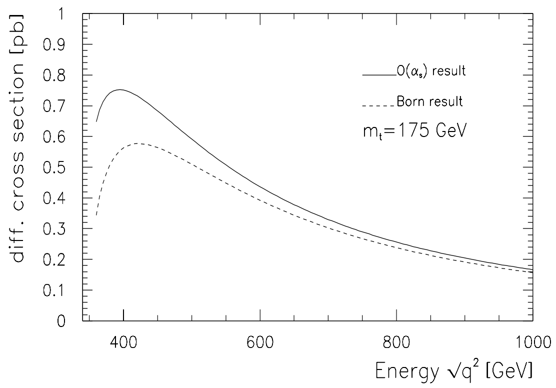

Figure 3: Energy dependence of $O(1)$ and $O(\alpha_s)$ cross section in $e^+e^- \to t\bar{t}(g)$

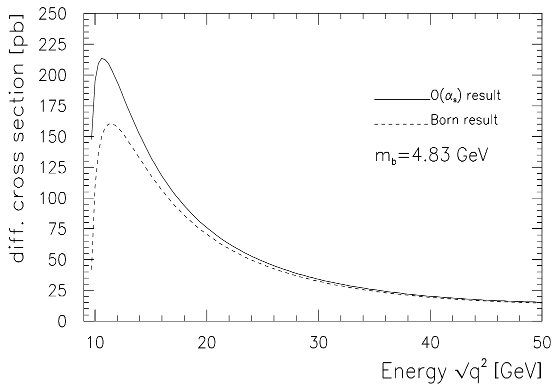

Figure 4: Energy dependence of $O(1)$ and $O(\alpha_s)$ cross section in $e^+e^- \to b\bar{b}(g)$.

totic limiting value $\langle P^{\ell\ell}\rangle = -1$ due to the fact that the corrections to the leading term are quadratic in the ratio $m/\sqrt{q^2}$. Contrary to this the $O(\alpha_s)$ curve remains below the naive limiting value of -1. From the limiting formula

$$\langle P^{\ell\ell}\rangle = -\frac{1+\dfrac{\alpha_s}{\pi} - \left[\dfrac{4\alpha_s}{3\pi}\right]}{1+\dfrac{\alpha_s}{\pi}} \tag{20}$$

one concludes that a large part of the deviation is made up by the anomalous contributions. For example, at the position of the Z pole the limiting value of the anomalous contribution to $\langle P^{\ell\ell}\rangle$ amounts to $\langle P^{\ell\ell}\rangle(anom) = 0.048$ ($\alpha_s(m_Z) = 0.118$). From the full calculation one finds $\langle P^{\ell\ell}\rangle$ $(Born) = -0.996$ and $\langle P^{\ell\ell}\rangle(\alpha_s) = -0.964$. Thus the deviation of $\langle P^{\ell\ell}\rangle$ from its naive value of $\langle P^{\ell\ell}\rangle = -1$ can be seen to arise to a large part from the anomalous contribution.

Fig. 3 and Fig. 4 show the cross section for the $t\bar{t}$ and $b\bar{b}$ production respectively, the $O(\alpha_s)$ correction enhances the cross section near threshold.

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