

$O(\alpha_s)$ corrections to longitudinal and transverse W -bosons in top quark decays

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ABSTRACT: In this talk we consider the $O(\alpha_s)$ radiative corrections to the decay of an unpolarized top quark into a bottom quark and a W gauge boson where the helicities of the W are specified as transverse-plus, transverse-minus and longitudinal. The $O(\alpha_s)$ radiative corrections lower the normalized longitudinal rate Γ_L/Γ by 0.8%. We find that the normalized transverse-plus rate Γ_+/Γ , which vanishes at the Born term level for $m_b \rightarrow 0$, receives radiative correction contributions at the sub-percent level. Our results are discussed in the light of recent measurements of the helicity content of the W in top quark decays by the CDF Collaboration.

1. Introduction

The CDF Collaboration has recently published the results of a first measurement of the helicity content of the W gauge boson in top quark decays [1]. Their results are

$$\Gamma_L/\Gamma = 0.91 \pm 0.37(stat) \pm 0.13(syst) \quad (1.1)$$

$$\Gamma_+/\Gamma = 0.11 \pm 0.15 \quad (1.2)$$

where Γ_L and Γ_+ denote the rates into the longitudinal and transverse-plus polarization state of the W -boson and Γ is the total rate.

The errors on this measurement are still rather large but will be much reduced when larger data samples become available in the future from TEVATRON RUN II, and, at a later stage, from the LHC. Optimistically the measurement errors can eventually be reduced to the (1–2)% level [2]. If such a level of accuracy can in fact be reached it is important to discuss the radiative corrections to the different helicity rates [3, 4] considering the fact that the $O(\alpha_s)$ radiative corrections to the total width Γ are rather large ($\approx -9.4\%$) [5, 6, 7, 8, 9]. The transverse-plus rate Γ_+ is particularly interesting in this regard.

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Simple helicity considerations show that Γ_+ vanishes at the Born term level in the $m_b = 0$ limit. A nonvanishing transverse-plus rate could arise from i) $m_b \neq 0$ effects, ii) $O(\alpha_s)$ radiative corrections due to gluon emission, or from iii) non Standard Model $t \rightarrow b$ currents. As we shall show in this talk the $O(\alpha_s)$ and the $m_b \neq 0$ corrections to the transverse-plus rate occur only at the sub-percent level. It is safe to say that if top quark decays reveal a violation of the Standard Model ($V - A$) current structure which exceeds the 1% level, the violations must have a non Standard Model origin.

2. Angular decay distribution

Let us begin by writing down the angular decay distribution for the decay process $t \rightarrow X_b + W^+$ followed by $W^+ \rightarrow l^+ + \nu_l$ (or by $W^+ \rightarrow \bar{q} + q$). For unpolarized top decay the angular decay distribution is determined by two transverse components (transverse-plus and transverse-minus) and the longitudinal component of the W -boson. One has [3]

$$\frac{d\Gamma}{d\cos\theta} = \frac{3}{8}(1 + \cos\theta)^2\Gamma_+ + \frac{3}{8}(1 - \cos\theta)^2\Gamma_- + \frac{3}{4}\sin^2\theta\Gamma_L. \quad (2.1)$$

Integrating over $\cos\theta$ one recovers the total rate

$$\Gamma = \Gamma_+ + \Gamma_- + \Gamma_L. \quad (2.2)$$

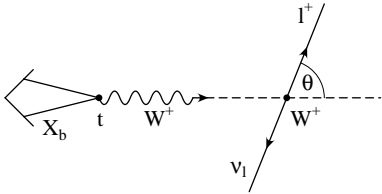


Figure 1: Definition of the polar angle θ

We describe the angular decay distribution in cascade fashion, i.e. the polar angle θ is measured in the W rest frame where the lepton pair or the quark pair emerges back-to-back. The angle θ denotes the polar angle between the W^+ momentum direction and the antilepton l^+ (or the antiquark \bar{q}) (see Fig. 1). The various contributions in (2.1) are reflected in the shape of the lepton energy spectrum in the rest frame of the top quark. From the angular factors in (2.1) it is clear that the contribution of Γ_+ makes the lepton spectrum harder while Γ_- softens the spectrum where the hardness or softness is gauged relative to the longitudinal contribution. The only surviving contribution in the forward direction $\theta = 0$ comes from Γ_+ . The fact that Γ_+ is predicted to be quite small implies that the lepton spectrum will be soft. The CDF measurement of the helicity content of the W^+ in top decays was in fact done by fitting the values of the helicity rates to the shape of the lepton's energy spectrum.

The angular decay distribution of the antitop decay $\bar{t} \rightarrow X_{\bar{b}} + W^-$ followed by $W^- \rightarrow l^- + \bar{\nu}_l$ (or by $W^- \rightarrow q + \bar{q}$) can be obtained from the angular decay distribution (2.1) by the substitution $(1 + \cos\theta)^2 \leftrightarrow (1 - \cos\theta)^2$. The polar angle θ is now defined with regard to the lepton l^- (or the quark) direction.

3. Born term results

The Born term contribution is shown in Fig. 2a. We use the scaled masses $x = m_W/m_t$, $y = m_b/m_t$ and the Källén-type function $\lambda = 1 + x^4 + y^4 - 2x^2y^2 - 2x^2 - 2y^2$. Including the full

m_b -dependence, the Born term rate is given by

$$\Gamma_0 = \frac{G_F m_W^2 m_t}{8\sqrt{2}\pi} |V_{tb}|^2 \sqrt{\lambda} \times \frac{(1-y^2)^2 + x^2(1-2x^2+y^2)}{x^2}. \quad (3.1)$$

The partial helicity rates are given in terms of the Born term rate. One has

$$\begin{aligned} \Gamma_L/\Gamma_0 &= \frac{(1-y^2)^2 - x^2(1+y^2)}{(1-y^2)^2 + x^2(1-2x^2+y^2)} \\ &= \frac{1}{1+2x^2} + \dots \end{aligned} \quad (3.2)$$

$$\begin{aligned} \Gamma_+/\Gamma_0 &= \frac{x^2(1-x^2+y^2-\sqrt{\lambda})}{(1-y^2)^2 + x^2(1-2x^2+y^2)} \\ &= y^2 \frac{2x^2}{(1-x^2)^3(1+2x^2)} + \dots \end{aligned} \quad (3.3)$$

$$\begin{aligned} \Gamma_-/\Gamma_0 &= \frac{x^2(1-x^2+y^2+\sqrt{\lambda})}{(1-y^2)^2 + x^2(1-2x^2+y^2)} \\ &= \frac{2x^2}{1+2x^2} + \dots \end{aligned} \quad (3.4)$$

In Eqs. (3.2–3.4) we have also listed the leading components of the small y^2 expansion of the Born term rate ratios. As Eq. (3.3) shows and as already remarked on before, the transverse-plus Born term contribution vanishes in the $m_b = 0$ limit. Note that the leading contribution to the transverse-plus rate is proportional to $(m_b/m_t)^2$ and not proportional to $(m_b/m_W)^2$ as stated in Ref. [1].

The $m_b \neq 0$ effects are quite small. Using $m_t = 175$ GeV, $m_W = 80.419$ GeV, and a pole mass of $m_b = 4.8$ GeV [10], one finds that Γ_0 and Γ_L/Γ_0 decrease by 0.266% and 0.091%, resp. when going from $m_b = 0$ to $m_b = 4.8$ GeV, and Γ_-/Γ_0 increases by 0.094%. The leakage into the transverse-plus rate ratio Γ_+/Γ_0 from bottom mass effects is a mere 0.036%.

4. $O(\alpha_s)$ radiative corrections

The $O(\alpha_s)$ corrections are determined by the one-loop vertex correction shown in Fig. 2b and the gluon emission graphs shown in Figs. 2c and 2d. The one-loop results have already been known for quite some time [11, 12] and will not be discussed any further in this talk.

We do want to make a few technical remarks about how the tree-graph integration was done.

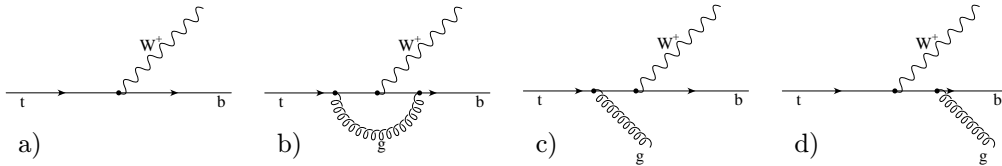


Figure 2: Leading order Born term contribution (a) and $O(\alpha_s)$ contributions (b,c,d) to $t \rightarrow b + W^+$.

We use a gluon mass to regularize the IR singularity. Concerning the collinear singularity we have kept the full bottom mass dependence in our calculation and have only set the bottom mass to zero at the very end. We have thus effectively used a mass regulator to regularize the collinear singularity. The tree-graph integration has to be done over two-dimensional phase space. As phase space variables we use the gluon energy k_0 and the W energy q_0 . The IR behaviour of the hadronic tree-graph matrix element $W_{\mu\nu}(q_0, k_0)$ was improved by subtracting from it the soft-gluon contribution $G_{\mu\nu}(q_0, k_0)$ which was then added again according to the prescription

$$W_{\mu\nu}(q_0, k_0) = (W_{\mu\nu}(q_0, k_0) - G_{\mu\nu}(q_0, k_0)) + G_{\mu\nu}(q_0, k_0) \quad (4.1)$$

The first piece $(W_{\mu\nu}(q_0, k_0) - G_{\mu\nu}(q_0, k_0))$ has thereby been rendered IR finite and can be integrated without a gluon mass regulator which considerably simplifies the phase space integration. The IR singularity resides in the soft gluon piece $G_{\mu\nu}(q_0, k_0)$ which is, however, simple and universal and can be easily integrated. In fact the soft gluon contribution factorizes into the Born term contribution $B_{\mu\nu}$ and a universal soft gluon factor $S(q_0, k_0)$ according to

$$G_{\mu\nu}(q_0, k_0) = B_{\mu\nu} \cdot S(q_0, k_0) \cdot \alpha_s. \quad (4.2)$$

The Born term contribution $B_{\mu\nu}$ is given by

$$B^{\mu\nu} = 8(p_t^\nu p_b^\mu + p_t^\mu p_b^\nu - g^{\mu\nu} p_t \cdot p_b + i\epsilon^{\mu\nu\alpha\beta} p_{b,\alpha} p_{t,\beta}), \quad (4.3)$$

while the soft-gluon factor in (4.2) has the standard form

$$S(q_0, k_0) = \frac{m_t^2}{(p_t k)^2} - \frac{2p_t p_b}{(p_t k)(p_b k)} + \frac{m_b^2}{(p_b k)^2}. \quad (4.4)$$

Note that the tensor structure carrying the spin information of the produced W -boson has been

factored out and is now entirely contained in the Born term factor $B_{\mu\nu}$. Since the Born term factor does not depend on the phase space variables, the phase space integration needs to be done only with respect to the soft gluon factor $S(q_0, k_0)$ and thus needs to be done only once irrespective of the polarisation of the W -boson. Needless to say that this is a very welcome simplifying feature of the above subtraction procedure.

This is different for the phase space integration of the IR-finite $(W_{\mu\nu}(q_0, k_0) - G_{\mu\nu}(q_0, k_0))$ where the integration has to be done separately for each polarization state of the W -boson. To do the necessary two-dimensional phase space integrations in analytical form is somewhat involved. In particular the integrations are more difficult than those needed for the total rate calculation which has already been done some time ago [5, 6, 7, 8, 9]. The complicating feature can be best appreciated by discussing how one obtains the various helicity components of the W -boson.

We chose to use covariant projectors to project out the various helicity components of the W -boson. For the total rate one has the familiar form

$$\mathbb{P}_{U+L}^{\mu\nu} = -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_W^2} \quad (4.5)$$

where we have added the label ($U+L$) for added emphasis. The projector (4.5) can be seen to reduce to the appropriate three-dimensional form δ_{ij} in the W rest system. The longitudinal helicity rate is obtained with the help of the projector

$$\mathbb{P}_L^{\mu\nu} = \frac{m_W^2}{m_t^2} \frac{1}{|\vec{q}|^2} \left(p_t^\mu - \frac{p_t \cdot q}{m_W^2} q^\mu \right) \left(p_t^\nu - \frac{p_t \cdot q}{m_W^2} q^\nu \right). \quad (4.6)$$

The transverse-plus and transverse-minus helicity projectors are obtained in an indirect way by first considering the sum and the difference of the transverse-plus and transverse-minus projectors. The sum is labelled by the index U

(unpolarized-transverse), and the corresponding projector is obtained from $\mathbb{P}_U^{\mu\nu} = \mathbb{P}_{U+L}^{\mu\nu} - \mathbb{P}_L^{\mu\nu}$. The difference of the transverse-plus and transverse-minus helicity projectors is labelled by the index F (forward-backward) projector which is given by

$$\mathbb{P}_F^{\mu\nu} = -\frac{1}{m_t} \frac{1}{|\vec{q}|} i\epsilon^{\mu\nu\alpha\beta} p_{t,\alpha} q_{\beta}. \quad (4.7)$$

The transverse-plus and transverse-minus projectors are thus determined by the linear combinations $\mathbb{P}_{\pm}^{\mu\nu} = \frac{1}{2}(\mathbb{P}_U^{\mu\nu} \pm \mathbb{P}_F^{\mu\nu})$.

The inverse powers of the magnitude $|\vec{q}| = \sqrt{q_0^2 - m_W^2}$ of the three-momentum of the W -boson appear in the projectors for normalization reasons. It is mainly because of the additional factors of $|\vec{q}|^{-n}$ in the helicity rates that makes their phase space integration technically more involved than the total rate integration since new classes of phase space integrals appear.

Our final results are presented for the $m_b = 0$ limit. We add together the Born term, the one-loop and the tree-graph contribution. The results are taken from Ref. [3]. We divide out the total $m_b = 0$ Born term rate and denote the scaled rates $\hat{\Gamma}_i = \Gamma_i/\Gamma_0$ ($i = U + L, L, +, -$) by a hat symbol. The two transverse helicity rates $i = +, -$ are obtained from the $i = U, F$ rates given in [3] by taking the linear combinations $\Gamma_{\pm} = \frac{1}{2}(\Gamma_U \pm \Gamma_F)$ as discussed before. One has

$$\hat{\Gamma} = 1 + \frac{\alpha_s}{2\pi} C_F \frac{x^2}{(1-x^2)^2(1+2x^2)} \times \left\{ \begin{aligned} &\frac{(1-x^2)(5+9x^2-6x^4)}{2x^2} \\ &-4(1+x^2)(1-2x^2)\ln(x) \\ &-\frac{(1-x^2)^2(5+4x^2)}{x^2}\ln(1-x^2) \\ &-\frac{4(1-x^2)^2(1+2x^2)}{x^2} \left(\ln(x)\ln(1-x^2) + \frac{\pi^2}{6} \right) \\ &-\frac{8(1-x^2)^2(1+2x^2)}{x^2} (\text{Li}_2(x) + \text{Li}_2(-x)) \end{aligned} \right\} \quad (4.8)$$

$$\hat{\Gamma}_L = \frac{1}{1+2x^2} + \frac{\alpha_s}{2\pi} C_F \frac{x^2}{(1-x^2)^2(1+2x^2)} \times \left\{ \begin{aligned} &\frac{(1-x^2)(5+47x^2-4x^4)}{2x^2} \\ &-\frac{(1+5x^2+2x^4)}{x^2} \frac{2\pi^2}{3} + 16(1+2x^2)\ln(x) \end{aligned} \right\}$$

$$\left. \begin{aligned} &-\frac{3(1-x^2)^2}{x^2}\ln(1-x^2) \\ &-2(1-x^2)^2\frac{2-x+6x^2+x^3}{x^2}\ln(1-x)\ln(x) \\ &-2(1+x)^2\frac{(2+x+6x^2-x^3)}{x^2}\ln(x)\ln(1+x) \\ &-2(1-x)^2\frac{(4+3x+8x^2+x^3)}{x^2}\text{Li}_2(x) \\ &-2(1+x)^2\frac{(4-3x+8x^2-x^3)}{x^2}\text{Li}_2(-x) \end{aligned} \right\} \quad (4.9)$$

$$\hat{\Gamma}_+ = \frac{\alpha_s}{2\pi} C_F \frac{x^2}{(1-x^2)^2(1+2x^2)} \times \left\{ \begin{aligned} &-\frac{1}{2}(1-x)(25+5x+9x^2+x^3) \\ &+(7+6x^2-2x^4)\frac{\pi^2}{3} - 2(5+7x^2-2x^4)\ln(x) \\ &-2(1-x^2)(5-2x^2)\ln(1+x) \\ &-\frac{(1-x)^2}{x}(5+7x^2+4x^3)\ln(x)\ln(1-x) \\ &+\frac{(1+x)^2}{x}(5+7x^2-4x^3)\ln(x)\ln(1+x) \\ &-\frac{(1-x)^2}{x}(5+7x^2+4x^3)\text{Li}_2(x) \\ &+\frac{1}{x}(5+10x+12x^2+30x^3-x^4-12x^5)\text{Li}_2(-x) \end{aligned} \right\} \quad (4.10)$$

$$\hat{\Gamma}_- = \frac{2x^2}{1+2x^2} + \frac{\alpha_s}{2\pi} C_F \frac{x^2}{(1-x^2)^2(1+2x^2)} \times \left\{ \begin{aligned} &-\frac{1}{2}(1-x)(13+33x-7x^2+x^3) \\ &+(3+4x^2-2x^4)\frac{\pi^2}{3} - 2(5+7x^2-2x^4)\ln(x) \\ &-2\frac{(1-x^2)^2(1+2x^2)}{x^2}\ln(1-x) \\ &-2\frac{(1-x^2)(1-4x^2)}{x^2}\ln(1+x) \\ &-\frac{(1-x)^2}{x}(5+7x^2+4x^3)\ln(x)\ln(1-x) \\ &+\frac{(1+x)^2}{x}(5+7x^2-4x^3)\ln(x)\ln(1+x) \\ &-\frac{(1-x)^2}{x}(5+3x)(1+x+4x^2)\text{Li}_2(x) \\ &+\frac{1}{x}(5+2x+12x^2+6x^3-x^4-4x^5)\text{Li}_2(-x) \end{aligned} \right\} \quad (4.11)$$

The expressions for the rates contain the usual logarithmic and dilogarithmic factors that appear in one-loop radiative correction calculations. As expected, the transverse-plus rate becomes

non-zero only at the $O(\alpha_s)$ level. Note that the entire $O(\alpha_s)$ contribution to the transverse-plus rate comes from the tree-graph contribution since the one-loop graph does not contribute to the transverse-plus rate for chirality reasons. As expected, the longitudinal rate becomes dominant in the high energy limit as $m_t \rightarrow \infty$ for both the Born term and the $O(\alpha_s)$ corrections.

5. Numerical results

We are now in a position to discuss our numerical results. Our input values are $m_t = 175$ GeV and $m_W = 80.419$ GeV, as before. For the strong coupling constant we use $\alpha_s(m_t) = 0.107$ which was evolved downward from $\alpha_s(m_Z) = 0.1175$. Our numerical results are presented in terms of the hatted helicity rates $\hat{\Gamma}_i = \Gamma_i/\Gamma_0$ ($i = U + L, L, +, -$) introduced in Sec. 4. In order to be able to quickly assess the percentage changes induced by the $O(\alpha_s)$ corrections, we have factored out the Born term helicity rates (when applicable) from the $O(\alpha_s)$ results. One has

$$\hat{\Gamma} = 1 - 0.0938, \quad (5.1)$$

$$\hat{\Gamma}_L = 0.703(1 - 0.104), \quad (5.2)$$

$$\hat{\Gamma}_+ = 0.001018, \quad (5.3)$$

$$\hat{\Gamma}_- = 0.297(1 - 0.0720), \quad (5.4)$$

The radiative corrections to the longitudinal and transverse-minus rates are sizeable where the radiative correction to the longitudinal rate is larger. The radiative corrections lower the normalized longitudinal rate Γ_L/Γ by 0.8% and increase the normalized transverse-minus rate Γ_-/Γ by 2.0%. The radiative correction to the transverse-plus rate is quite small. For the normalized transverse-plus rate Γ_+/Γ we obtain a mere 0.10% which is only marginally larger than the value of 0.036% obtained from the Born term level $m_b \neq 0$ effects discussed in Sec. 3.

6. Summary and conclusions

In this talk we have presented results on the $O(\alpha_s)$ radiative corrections to the three helicity rates in unpolarized top quark decays which can be determined from doing an angular analysis or from an analysis of the shape of the lepton spectrum. While the radiative corrections to

the unnormalized transverse-minus and longitudinal rate are sizable ($\approx 6 - 10\%$), the radiative corrections to the normalized helicity rates are smaller ($\approx 1 - 2\%$). The radiative correction to the transverse-plus rate is very small. The measurements of the helicity rates by the CDF Collaboration can be seen to be fully compatible with the predictions of the Standard Model. The errors on these measurements are, however, too large to allow one to meaningfully compare the present measurements with quantum effects brought in by QCD radiative corrections. There is hope that this will change in the future.

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