

# The Supergravity Dual of N=1 Super Yang-Mills

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**ABSTRACT:** We present an exact supersymmetric solution of 5-dimensional supergravity. We show that it represents the RG flow from N=4 SYM deformed with a mass term for the fermions to an IR N=1 super Yang-Mills theory. We discuss the properties of the solution and we briefly comment on the fate of the singularity. We also compare the supergravity results with the expectations of an N=1 SYM at strong coupling.

## 1. Introduction

The AdS/CFT can be interestingly extended to the analysis of non-conformal field theory. For example, it is possible to construct the supergravity duals of RG flows [1]-[14]. RG flows in a  $d$ -dimensional QFT correspond to type II or M-theory supergravity backgrounds that break the full  $O(d, 2)$  invariance while preserving at least  $d$ -dimensional Poincaré invariance, and that are asymptotically AdS. Many results have been obtained upon reduction to a  $d + 1$ -dimensional effective theory, where the RG flow can be studied in terms of a theory of scalar fields coupled to gravity. In this picture flows between CFTs are given by kinks solutions interpolating between two  $\text{AdS}_{d+1}$  vacua, while flow to non-conformal IR field theories are given by solutions with only one asymptotic AdS region and approaching infinity on the other side.

Flows to non-conformal theories are actually the most problematic. Since it is very rare to have the full analytical solution, it is not always clear what is their QFT interpretation: deformations of an UV fixed point versus the same theory in a different vacuum [15, 16]. Another important point is the reliability of these solutions, which all present a (typically naked) singularity in the IR region of the flow. Since the curvature and the kinetic terms for the scalars typi-

cally diverge in the IR region, large corrections to supergravity may be expected. By itself, this does not immediately implies that all the other non-conformal solutions should be discarded.

It is therefore interesting to study examples where comparisons with the field theory results can be made.

Here we focus on the flow from N=4 SYM to pure N=1 SYM<sup>1</sup>. Despite the singularity, supergravity results are in good qualitative agreement with quantum field theory expectations: quarks confine, monopoles are screened, and there is a gaugino condensate.

## 2. The flow to N=1 SYM

Consider a deformation of N=4 Super Yang-Mills theory with a supersymmetric mass term for the three fermions in the chiral N=1 multiplets. In N=1 notations, this is a mass term for the three chiral superfields  $X_i$

$$\int d^2\theta m_{ij} \text{Tr } X_i X_j + \text{c.c.}, \quad (2.1)$$

where  $m_{ij}$  is a complex, symmetric matrix.

The theory flows in the IR to pure N=1 Yang-Mills, which confines. To obtain the standard N=1 pure Yang-Mills with fixed scale  $\Lambda$ , we need a fine tuning of the UV parameters, in which

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<sup>1</sup>A more general and complete discussion of the  $5d$  approach to RG flows can be found in A. Zaffaroni's talk at this conference.

the mass  $m$  diverges while the 't Hooft coupling constant,  $x$ , goes to zero as an (inverse) logarithm of  $m$ . This is outside the regime of validity of supergravity, which requires a large  $x$ . We can think of  $m$  as a regulator for N=1 SYM. When embedded in N=4 SYM, the theory is finite. To get a well defined N=1 SYM, we remove the cut-off ( $m \rightarrow \infty$ ) with a fine tuning of the coupling ( $x(m) \rightarrow 0$ ). However, if we use supergravity, we are in the large  $x$  regime. The massive modes have a mass comparable with the scale of N=1 SYM and they do not decouple. We can think of this as a theory with an ultraviolet cut-off. A good analogy is with lattice gauge theory.  $1/m$  corresponds to the lattice spacing. The continuum limit is obtained with a fine tuning  $a \rightarrow 0, g(a) \rightarrow 0$ . However we can study the lattice theory at strong coupling, far from the continuum limit. A standard computation at strong coupling (by Wilson) gives the area law. We are just doing analogous computations with supergravity. Qualitative features of the theory should hold also at strong coupling.

We want now to construct the five-dimensional supergravity solution corresponding to this deformation. The 5-dimensional action for the scalars [17]

$$L = \sqrt{-g} \left[ -\frac{R}{4} - \frac{1}{24} \text{Tr} (U^{-1} \partial U)^2 + V(U) \right], \quad (2.2)$$

is written in terms of a  $27 \times 27$  matrix  $U$ , transforming in the fundamental representation of  $E_6$  and parametrising the coset  $E_6/USp(8)$ . In a unitary gauge,  $U$  can be written as  $U = e^X$ ,  $X = \sum_a \lambda_a T_a$ , where  $T_a$  are the generators of  $E_6$  that do not belong to  $USp(8)$ . This matrix has exactly 42 real independent parameters, which are the scalars of the supergravity theory. They transform in the following  $SO(6)$  representations:  $\underline{10}_c$ ,  $\underline{20}$ , and  $\underline{1}_c$ . The supersymmetric mass term for the chiral multiplets,  $m_{ij}$ , transforms as the  $\underline{6}$  of  $SU(3) \in SU(4)$  ( $SO(6) \sim SU(4)$ ), and the corresponding supergravity mode appears in the decomposition of the  $\underline{10} \rightarrow \underline{1} + \underline{6} + \underline{3}$  of  $SU(4)$  under  $SU(3) \times U(1)$ . The term  $\underline{1}$  in this decomposition corresponds instead to the scalar  $\sigma$  dual to the gaugino condensate in N=1 SYM. In principle, a generic non-zero VEV for  $m_{ij}$  will induce non-

zero VEVs for other scalars as well, due to the existence of linear couplings of  $m$  to other fields in the potential. However, if we further impose  $SO(3)$  symmetry, by taking  $m_{ij}$  proportional to the identity matrix, a simple group theory exercise shows that all the remaining fields can be consistently set to zero. This is true also if we consider a two-parameter Lagrangian depending on both  $m$  and  $\sigma$ . This felicitous circumstance makes an apparently intractable problem very simple and exactly solvable.

The actual computation is reported in [11]. The result for the action for  $m$  and  $\sigma$  (the reason why we are considering both modes will be clear very soon) is

$$L = \sqrt{-g} \left\{ -\frac{R}{4} + \frac{1}{2} (\partial m)^2 + \frac{1}{2} (\partial \sigma)^2 + \right. \\ \left. -\frac{3}{8} \left[ \left( \cosh \frac{2m}{\sqrt{3}} \right)^2 + 4 \cosh \frac{2m}{\sqrt{3}} \cosh 2\sigma \right. \right. \\ \left. \left. - (\cosh 2\sigma)^2 + 4 \right] \right\}. \quad (2.3)$$

In ref. [8, 18] the conditions for a supersymmetric flow were found. For a supersymmetric solution, the potential  $V$  can be written in terms of a superpotential  $W$  as

$$V = \frac{1}{8} \sum_{a=1}^n \left| \frac{\partial W}{\partial \lambda_a} \right|^2 - \frac{1}{3} |W|^2, \quad (2.4)$$

where  $W$  is one of the eigenvalues of the tensor  $W_{ab}$  defined in [17]. As usual, a solution for which the fermionic shifts vanish, automatically satisfies the equations of motion. Moreover, this shortcut reduces the second order equations to first order ones

$$\dot{\lambda}_a = \frac{1}{2} \frac{\partial W}{\partial \lambda_a}, \quad (2.5)$$

$$\dot{\phi} = -\frac{1}{3} W. \quad (2.6)$$

The action has the supersymmetric form (2.4) with  $W = -\frac{3}{4} \left( \cosh \frac{2m}{\sqrt{3}} + \cosh 2\sigma \right)$ . The first order equations (2.6) read

$$\dot{\phi} = \frac{1}{2} \left( 1 + \cosh \frac{2m}{\sqrt{3}} \right) \quad (2.7)$$

$$\dot{m} = -\frac{\sqrt{3}}{2} \sinh \frac{2m}{\sqrt{3}}, \quad (2.8)$$

$$\dot{\sigma} = -\frac{3}{2} \sinh 2\sigma. \quad (2.9)$$

One interesting feature of the solution is that the equations can be analytically solved. To the best of our knowledge, there is only another example of analytically solvable flow, describing the Coulomb branch of N=4 SYM [9]. The solution in our case is:

$$\begin{aligned} \phi(y) &= \frac{1}{2} \log[2 \sinh(y - C_1)] + \\ &+ \frac{1}{6} \log[2 \sinh(3y - C_2)], \end{aligned} \quad (2.10)$$

$$m(y) = \frac{\sqrt{3}}{2} \log \left[ \frac{1 + e^{-(y-C_1)}}{1 - e^{-(y-C_1)}} \right], \quad (2.11)$$

$$\sigma(y) = \frac{1}{2} \log \left[ \frac{1 + e^{-(3y-C_2)}}{1 - e^{-(3y-C_2)}} \right]. \quad (2.12)$$

Here  $y$  is the fifth coordinate of AdS<sub>5</sub>, which we interpret as an energy scale [19, 20]:  $y \rightarrow \infty$  corresponds to the UV regime while  $y \rightarrow -\infty$  to the IR. The metric has a singularity at  $y = C_1$  ( $A = 1/2$ )

$$ds^2 = dy^2 + |y - C_1| dx^\mu dx_\mu. \quad (2.13)$$

Around this point  $m$  behaves as

$$m \sim -\frac{\sqrt{3}}{2} \log(y - C_1) + \text{const.} \quad (2.14)$$

Here we assumed that  $C_2 \leq 3C_1$ , so that at the point where  $m$  is singular,  $\sigma$  is still finite.

## 2.1 Properties of the solution

Let us discuss the qualitative properties of the N=1 SYM solution.

It is easy to see that the solution corresponds to a true deformation of the gauge theory. Indeed,  $m$  approaches the boundary in the UV ( $y \rightarrow \infty$ ) as  $m \sim e^{-y}$ , which is the required behaviour of a deformation<sup>2</sup>. On the other hand,  $\sigma$  has the UV behaviour appropriate for a condensate  $\sigma \sim e^{-3y}$ . Let us stress that this behaviour is enforced by the requirement of N=1 supersymmetry along the flow. The interpretation of the solution is therefore the following: upon perturbation with a mass term for the three chiral

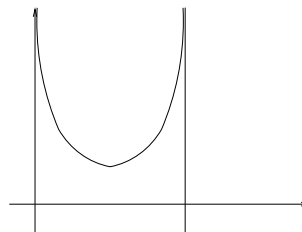
<sup>2</sup>The field theory interpretation of a supergravity solution depends on its asymptotic UV behaviour. Solutions asymptotic to  $e^{-(4-\Delta)y}$  describe deformations of a CFT with the operator  $O(x)$ , while solutions asymptotic to  $e^{-\Delta y}$  correspond to the same theory in a different vacuum where the operator  $O(x)$  has a non-zero VEV [15, 16].

fields, the N=4 SYM theory flows in the IR to pure N=1 SYM in a vacuum with a non-zero gaugino condensate. The existence of a gaugino condensate is one of the QFT expectations for N=1 SYM.

We also expect the gauge theory to exhibit confinement in the IR. We can easily compute a two-point function for a minimally-coupled scalar in the background with  $\sigma = 0$ . In our example, the Schroedinger potential is

$$V(z) = \frac{6 \cos(2z) + 9}{\sin^2(2z)}. \quad (2.15)$$

It is obvious from the figure, that there is mass gap and a discrete spectrum.



**Figure 1:** The potential for the N=1 SYM flow.

The AdS boundary is at  $z = 0$  and the singularity at  $z = \pi/2$ . The 2-point function for the massless scalar corresponding to  $F^2$  can be explicitly computed [21]:

$$\langle F^2(k) F^2(0) \rangle \sim k^2 (k^2 + 4) \text{Re} \psi(2 + ik). \quad (2.16)$$

It approaches the conformal expression  $k^4 \log k$  in the UV and it is analytic for small  $k$ , as appropriate for a confining theory. It has poles for  $M^2 = -k^2 = n^2$ ,  $n = 2, 3, \dots$ , corresponding to the  $F^2$  glueball states in the spectrum.

Despite the presence of a singularity that invalidates the supergravity approximation in the IR, the qualitative properties of the solution agree with the QFT expectations. There is however a disturbing point: our solution depends on two independent parameters  $C_1$  and  $C_2$ . The first one fixes the position of the singularity and it is related to the magnitude of the mass deformation. The second one is instead related to the magnitude of the gaugino condensate. We have a chirally-symmetric vacuum and, more disturbing, a continuous degeneracy of vacua with arbitrary small condensate. We certainly expect that the correct treatment of the singularity and its resolution in string theory fixes the relation between  $C_1$  and  $C_2$  in agreement with field theory expectations. We do not still know how

to resolve or deal with the singularity, therefore we limit ourself to a brief discussion of the QFT expectations and possible interpretations of the singularity.

## 2.2 QFT and string expectations

Strong coupling QFT results for N=1 SYM have been recently obtained and differ considerably from the weak coupling ones [22]. At weak coupling, spontaneous breaking of the chiral symmetry  $Z_N$  gives  $N$  vacua that only differ for the phase of the gaugino condensate  $\langle \lambda\lambda \rangle \sim e^{2\pi i k/N} \Lambda_{N=1}^3$ . In the large  $N$  limit, we obtain a *circle* of vacua. The magnitude of the gaugino condensate is fixed in terms of the SYM scale  $\Lambda_{N=1} \sim m e^{-1/3Ng^2}$ . At strong coupling instead, it was shown in [22] that there is, at least for  $\theta = 0$ , a distribution of vacua with condensate  $\langle \lambda\lambda \rangle \sim m^3 x^3 / j^2$ ,  $j = 1, 2, \dots$  with zero phase. The weakly coupled *circle* is lost, the condensate magnitude is not fixed and the vacua have an accumulation point at the origin (zero condensate). However, we notice that the structure of vacua found in [22] has many similarities with our supergravity result. As independently noticed in [23], it is tempting to identify the solution with  $C_2 = 3C_1$  with the  $j = 1$  vacuum in [22]. The other solutions with  $C_2 \leq 3C_1$  should correspond to the  $j \neq 0$  vacua. To see how the continuum of vacua in supergravity is reduced to a discrete numerable set, we should understand how to include string corrections in our computation. Notice that the solution with  $\sigma = 0$ , which is not appealing on the ground of weak coupling intuition, could be nevertheless used as a (reasonable?) approximation for the many vacua with small condensate at strong coupling.

The knowledge of the full 10 dimensional solution would greatly help in understanding the properties of the RG flow and in studying possible resolutions of the singularity. It may even happen that the singularity is an artifact of the dimensional reduction, that disappears in 10d. This happens, for example, in the case of the Coulomb branch of N=4 SYM [9], where the 10 dimensional background is just a regular continuous distribution of D3-branes. A ten-dimensional interpretation of the N=1 solution in terms of a background with also D5-branes has been pro-

posed in [24]. We only notice that the ingredients in this interpretation (D5 and NS-branes) have been independently suggested in [22] on the basis of the strong coupling QFT analysis. Finally, we mention that a mechanism for resolving singularities in distributions of branes which may help, after the 10d lifting, has been proposed in [25].

## 2.3 The Wilson loop

A complementary approach for checking confinement is the computation of a Wilson loop, which should manifest an area law behaviour. We need to minimise the action for a string whose endpoints are constrained on a contour  $C$  on the boundary. The detailed computation is reported in [6, 11]. In the coordinates used in those papers, the quark-antiquark energy reads

$$E = S/T = \int dx \sqrt{(\partial_x u)^2 + f(u)}. \quad (2.17)$$

where  $f(u) = T^2(u) e^{4\phi(u)}$ . The phase of the theory can be inferred by the IR behaviour of this function (see [6] for a review of the various cases).  $T(u)$  is the tension of the fundamental (in the case of a quark) or of the D1 string (monopole) in five dimensions, which in general is a non-trivial function of the scalar fields. The 5d N=8 gauged supergravity has an  $SL(2, Z)$  symmetry that allows to discriminate electric and magnetic strings. They should couple to the 5d antisymmetric tensors  $B_{\mu\nu}^{I\alpha}$ , transforming in the  $(\underline{6}, 2)$  of  $SO(6) \times SL(2, Z)$ . The  $SO(6)$  index should account for the orientation of the strings on the five-sphere, while the  $SL(2, Z)$  index should identify electric and magnetic quantities. On the basis of naive dimensional reduction from ten dimensions, the tensions can be read from the coefficients of the kinetic term for the antisymmetric tensors. In 10 dimensions, the tension of the fundamental string (or the D1-string) can be read from the coefficient of the kinetic term for the NS-NS (or R-R) antisymmetric tensor in the Lagrangian evaluated in the Einstein frame,

$$\frac{1}{T_{F1}^2} H_{NS-NS}^2 + \frac{1}{T_{D1}^2} H_{R-R}^2. \quad (2.18)$$

A simple Weyl rescaling shows that this property is valid also in the five-dimensional theory in the Einstein frame.

The kinetic terms for the anti-symmetric tensors can be computed for the N=1 SYM solution and behave asymmetrically in the  $SL(2, Z)$  indices [11]. The final result for the tensions  $T(u)$  of the fundamental strings and of the D1-strings are, respectively,

$$T_{F1}^2 = 4 \left( \cosh \frac{4m}{\sqrt{3}} + \cosh \frac{2m}{\sqrt{3}} \right), \quad (2.19)$$

$$T_{D1}^2 = 8 \left( \cosh \frac{m}{\sqrt{3}} \right)^2, \quad (2.20)$$

so that the asymptotic behaviour of the corresponding functions  $f(u)$  is

$$f_{(q\bar{q})}(u) \sim 1, \quad f_{(m\bar{m})}(u) \sim |u - C_1|. \quad (2.21)$$

It is easy to check that  $f_{(q\bar{q})}(u)$  is bounded from below. It follows that the energy  $E \geq cL$ , where  $L$  is the quark distance. It can be easily proven that it is in fact  $E = cL$ , implying an area law behaviour for the Wilson loop, as expected for a confining theory. The IR behaviour of  $f_{(m\bar{m})}(u)$  implies, on the other hand, that monopoles are screened (see [6] for a review).

There is an apparent contradiction in the previous reasoning. The  $5d$  dilaton is not running in our solution. If the  $10d$  dilaton were also constant, the tension for a fundamental string would be proportional to the tension of a D1-string and the same would be true also after dimensional reduction to 5 dimensions. The  $5d$  tensions would be then complicated functions of the scalars, but invariant under  $SL(2, Z)$ . We instead find an  $SL(2, Z)$  asymmetric result from the N=8 gauged supergravity evaluated along our solution. A possible way out is to assume that, against naive expectations, the  $10d$  dilaton is not constant. We do not know of any argument that rules out this possibility. Since we are not expert in reconstructing  $10d$  solutions from  $5d$  ones, we just limit ourselves to consider this option and perform some very preliminary check on the equations of motion.

The  $10d$  dilaton equation of motion is

$$\partial^2 \phi \sim G_{MNP} G^{MNP}. \quad (2.22)$$

Therefore, a non-vanishing anti-symmetric tensor is a source for the dilaton. We can perform a check on our solution at the linearised

level. Consider a generic fluctuation of the anti-symmetric tensor  $B_{ab} = f_I(y) Y_{[ab]}^{I\pm}$ . We refer to [26] for notations and useful equations. Here  $Y_{[ab]}^{I\pm}$ ,  $a, b = 1, \dots, 5$  are harmonic functions on the five-sphere, transforming in the representation  $I$  of  $SO(6)$ . They satisfy  $\epsilon_{abcde} \partial_c Y_{[de]} = \pm 2i(k+2) Y_{[ab]}$ , where  $k$  is an integer labelling the harmonic degree. It is then easy to check that

$$\partial^2 \phi \sim \frac{1}{3} ((\partial_y f)^2 - (k+2)^2 f^2) Y_{[ab]} Y_{[ab]} \quad (2.23)$$

In our case ( $I = \underline{10}$ )  $k = 1$ . Since we are considering a deformation of the UV fixed point,  $f \sim e^{-x}$ , we see that the dilaton must run. Notice that instead, considering a different vacuum of the UV theory, one has  $f \sim e^{-3x}$ , and the dilaton remains constant (at least at the first perturbative order).

We still need to check that  $Y_{[ab]} Y_{[ab]} \neq 0$ . There is at least one example where  $Y_{[ab]} Y_{[ab]} = 0$ : the  $SU(3) \times U(1)$  critical point of the N=8 supergravity, whose  $10d$  solution is explicitly known [27]. In the product  $\underline{10} \times \underline{10} = \underline{20} + \dots$ , only the indicated term contains scalar terms ( $SO(5) \in SO(6)$  singlets). It is easy to check that, decomposing  $\underline{10} = \underline{1} + \underline{3} + \underline{6}$  under  $SU(3) \times U(1)$ , the  $\underline{1}$  term (related to the  $SU(3) \times U(1)$  critical point) has vanishing square. The N=1 mass term  $\underline{6}$ , however, has non vanishing square.

This argument is certainly not a proof that the  $10d$  dilaton runs. However, we find this option appealing. A running of the  $10d$  dilaton would agree with an interpretation of our solution that includes branes others than the D3s. In many respects, the knowledge of the explicit  $10d$  solution would help us in understanding the system, from the constituent branes to the fate of the singularity. Using a D3-brane probe in the  $10d$  background we could also explicitly compute the running of the gauge coupling along the flow.

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