Form factors in charm meson semileptonic decays

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Abstract: Recent measurements of form factors in semileptonic charm meson decay are reviewed, and prospects for better measurements in the near future are discussed.

This is a review of the most recent measurements of form factors in charm meson decay, and of prospects for measurements in the immediate future. The data are summarized in table 1. Published results come mainly from Fermilab (FNAL) fixed-target experiments E687 and E791. FNAL E831 (FOCUS) is now analyzing a much larger data sample and is just starting to produce physics results. The CLEO detector, which has published results based on 1.7 fb^{-1} and 3 fb^{-1} of e^+e^- collisions, now has 11 fb^{-1} of data and will report charm form factors in the near future.

1. Semileptonic decay to pseudoscalar mesons

In the limit of zero lepton mass, decays such as $D^0 \to K^-\ell^+\nu$ and $D^0 \to \pi^-\ell^+\nu$ are governed by a single form factor $f_+(q^2)$, where q^2 is the 4-momentum transfer (or mass of the virtual W). A second form factor f_- is observable at small q^2 in semimuonic decay. It is traditional to parametrize form factor measurements with the dipole form $f(q^2) = f(0)/(1 - q^2/M^2_{pole})$, where the closest exchange pole with correct quantum numbers has $M_{pole} \approx$2.1-2.5 GeV. For heavy quark effective theory (HQET), the maximum kinematically allowed q^2 (q^2_{max}) is a more natural reference point.

The Cabibbo-suppressed (CS) mode $D^0 \to \pi^-\ell^+\nu$ is especially interesting because of its connection to the measurement of the CKM matrix element $|V_{bu}|$. The rate for $B^0 \to \pi^-\ell^+\nu$ is proportional to $|V_{bu}|^2 \cdot |FF|^2$, where FF is the form factor; thus FF must be known to get $|V_{bu}|^2$ from data. The rate for $D^0 \to \pi^-\ell^+\nu$ is proportional to $|V_{cd}|^2 \cdot |FF|^2$. V_{cd} is linked to the well-measured V_{us} by unitarity, and HQET relates the form factors for semileptonic B and D decays to light quarks.

It would be especially useful to have a measurement of the q^2 dependence of f_+, since the q^2 range is quite large, giving a good lever arm for slope determination. Note that $q^2_{max} = (M^2_D - m^2_\pi) = 2.98$ GeV^2 for $D^0 \to \pi^-\ell^+\nu$, but is only $q^2_{max} = (M^2_D - m^2_K) = 1.88$ GeV^2 for $D^0 \to K^-\ell^+\nu$. At the present time, this q^2 dependence is well-measured [?] for the Cabibbo-favored (CF) decay but unmeasured for the much harder CS decay.

1.1 $D^0 \to K^-\ell^+\nu$

The best measurement of form factors for $D^0 \to K^-\ell^+\nu$ is still that from FNAL E687 [?] in 1995. Results are based on 1897 muon events without a D^* tag. A cleaner sample of 427 events with D^* tag gives consistent results. E687 reports

$$f_+(0) = 0.71 \pm 0.03 \pm 0.03,$$

and the ratio

$$f_-(0)/f_+(0) = -1.3 \pm 3.5 \pm 0.6;$$

this ratio is expected to be small. Assuming the traditional q^2 dependence $f(q^2) = f(0)/(1 -$

Form factor measurement	Sample size
FNAL E687 (γ Be $\sim$200 GeV)	
$D^0 \to K^-\mu^+\nu$ (1995) [?]	1897 with no D^* tag 427 with D^* tag
$D^+ \to \overline{K}^{*0}\mu^+\nu$ (1993) [?]	900
$D_s^+ \to \phi\mu^+\nu$ (1994) [?]	90
FNAL E791 (π^-C, π^-Pt 500 GeV)	
$D^+ \to \overline{K}^{*0}e^+\nu$ (1998) [?]	3000
$D^+ \to \overline{K}^{*0}\mu^+\nu$ (1998) [?]	3000
$D_s^+ \to \phi e^+\nu$ (1999) [?]	144
$D_s^+ \to \phi\mu^+\nu$ (1999) [?]	127
CLEO (e^+e^-, $\sim$10.5 GeV)	
$D_s^+ \to \phi e^+\nu$ (1994) [?]	474
FNAL FOCUS: E831 (γ Be $\sim$200 GeV)	
$D^0 \to K^-\ell^+\nu$	*Expect [?] $\sim$40,000 with D^* tag*
$D^0 \to \pi^-\ell^+\nu$	*Expect [?] $\sim$5,000 with D^* tag*
$D^+ \to \overline{K}^{*0}\ell^+\nu$	*Expect [?] $\sim$60,000*
$D_s^+ \to \phi\ell^+\nu$	*Expect [?] $\sim$10,000*
$D^+ \to \rho^0\ell^+\nu$	*Expect $\sim$2,000 ? (My guess)*

Table 1: Recent and ongoing form factor measurements.

q^2/M^2_{pole}), E687 finds

$$M_{pole} = 1.87^{+0.11+0.07}_{-0.08-0.06}\ \text{GeV},$$

somewhat smaller than the expected value of $M_{pole} \approx 2.11$ GeV. It is worth pointing out that $D^0 \to K^-\ell^+\nu$ remains the *only* charm form factor with well-measured q^2 dependence.

1.2 Prospects for $D^0 \to \pi^-\ell^+\nu$

Figure 1 (from reference[?]) illustrates the difficulty of even a branching ratio measurement for the CS mode; the E687 [?] sample was 45 ± 13 electron events and 46 ± 12 muon events. In contrast to the CF case, a D^* tag absolutely must be used to reduce background. Even so, significant backgrounds remain: $D^0 \to K^-\ell^+\nu$ feedthrough comprises $\sim$ 50% of the D^* signal even after Cherenkov ID is applied, and combined feedthrough from the CS vector decays $D^0 \to K^{*-}\ell^+\nu$ and $D^0 \to \rho^-\ell^+\nu$ are each half as large as the $\pi^-\ell^+\nu$ signal. To make matters worse, cuts on the invariant mass of $h^-\ell^+$, which reduce feedthrough from vector decays, deplete the data at high q^2 where it is most valuable for getting form factor q^2 dependence.

FNAL E831 (FOCUS) will greatly improve this situation. They have shown [?] plots, based on 23% of their data sample, which extrapolate to 40,000 $D^0 \to K^-\ell^+\nu$ events with D^* tag in their final sample (compared to 427 comparable events from E687), and expect $\sim 5,000$ tagged $D^0 \to \pi^-\ell^+\nu$ events. With a sample of this size, FOCUS should be able to measure the q^2 dependence of the Cabibbo-suppressed mode – but it won't be easy.

2. Semileptonic decay to vector mesons

In the case of semileptonic D or D_s decay to a vector meson daughter, all strong interaction information is contained in the three form factors $A_1(q^2)$, $A_2(q^2)$ (axial-vector), and $V(q^2)$ (vector), plus a fourth form factor $A_3(q^2)$ which is measurable only at small q^2 and only in the muon case; $A_3(q^2)$ is additionally expected to be small. Experiments so far have quoted form factor ratios at $q^2 = 0$:

$$r_2 = A_2(0)/A_1(0),$$
$$r_V = V(0)/A_1(0),$$
$$r_3 = A_3(0)/A_1(0).$$

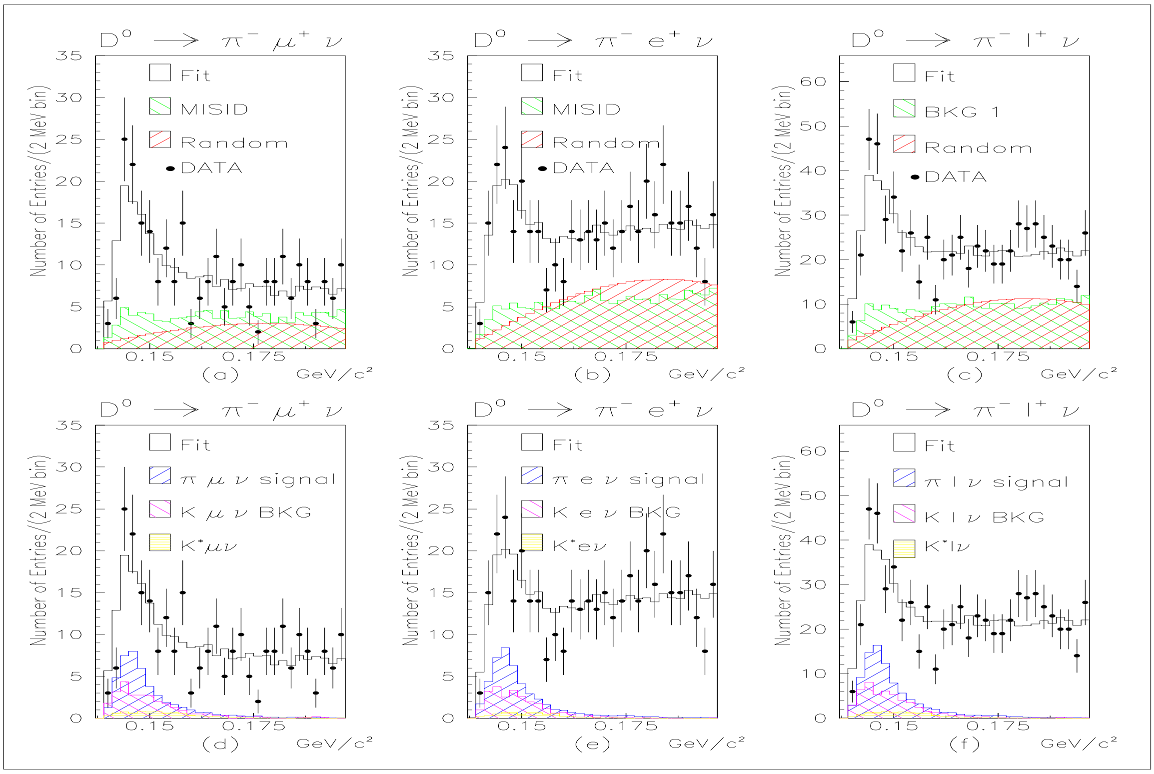

Figure 1: Extracting a signal for $D^0 \to \pi^- \ell^+ \nu$ (from reference[**?**]). D^* peaks in the muon (left), electron (center) and combined (right) modes. The top row shows the background contributions from sources other than semileptonic decay, while the bottom row shows the $\pi^- \ell^+ \nu$ signal and the feedthrough contributions from other semileptonic modes.

Further, a q^2 dependence

$$A, V(q^2) = f(0)/(1 - q^2/M_{A,V}^2)$$

is usually assumed, with pole masses

$$M_V = 2.1 \text{ GeV}, \quad M_A = 2.5 \text{ GeV}.$$

Information about form factors in vector decays is extracted from joint distributions in q^2 and the three decay angles shown in figure 2. Polar angles θ_l and θ_V characterize the decays of the virtual W and the vector meson in their respective rest frames; interference between helicity amplitudes affects the azimuthal angle χ between the two decay planes. The differential decay rate is a function of these angles and of helicity amplitudes, which in turn depend on q^2 and on the form factors. The dependence on angles and on q^2 is more complicated for the muon case: in addition to a kinematic suppression $(1 - q^2/m_\mu^2)$ near $q^2 = 0$ of the amplitudes present in the electron case, new terms in A_1, A_2 and V with different dependences on the decay angles become important at small q^2. If the fourth form factor A_3 is not negligibly small, still more terms in A_3 enter at small q^2. Since this different q^2 dependence for e and μ modes is a possible source of systematic error, it is certainly desirable to measure form factors for both modes in the same experiment.

2.1 $D^+ \to \overline{K}^{*0} \ell^+ \nu$

The well-measured K^* mode serves as a precision test bed for lattice QCD. FNAL E791 [**?**, **?**] now has 3,000 events in each of the e, μ modes; this is the first time both modes have been measured in the same experiment. Showing consistent form factor results in the e and μ modes is an essential test of systematics because of the q^2 complications described above.

As shown in the E791 data (figure 3), there are two distinct signatures for this decay mode:

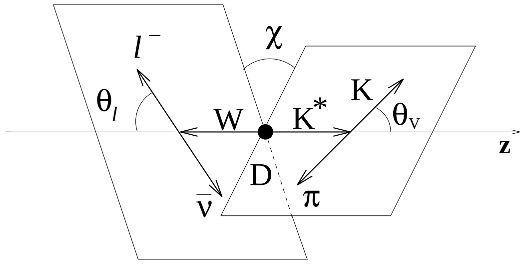

Figure 2: Definition of the decay angles θ_l, θ_V and χ used to extract form factors in D, D_s decays to vector daughters.

a K^* peak in $K^-\pi^+$ invariant mass; and a cusp at the mass of the charm parent in M_{min}, which is the invariant mass of the $K\pi\ell\nu$ invariant mass when the unmeasurable neutrino longitudinal momentum is ignored. Right-sign (RS) events are those in which K and ℓ have opposite charge, while wrong-sign (WS) events, a good measure of reconstruction background, have K and ℓ of the same charge. E791 has comparable numbers of events in the two decay modes: RS$\approx$ 3600, WS$\approx$ 600 in each.

E791 obtains consistent results for form factor ratios in the two channels:
for the electron mode,

$$r_2(0) = 0.71 \pm 0.08 \pm 0.09,$$

$$r_V(0) = 1.90 \pm 0.11 \pm 0.09;$$

for the muon mode,

$$r_2(0) = 0.75 \pm 0.08 \pm 0.09,$$

$$r_V(0) = 1.84 \pm 0.11 \pm 0.09.$$

The E791 result from the combined channels is

$$r_2(0) = 0.73 \pm 0.06 \pm 0.08,$$

$$r_V(0) = 1.87 \pm 0.08 \pm 0.07.$$

Figure 4 compares E791 single-variable decay distributions in q^2/q^2_{max}, $\cos\theta_l$, $\cos\theta_V$ and χ with Monte Carlo simulations using these form factor ratios. Note that all distributions except q^2 are very similar for the two modes, but that the muon mass effects described above have a significant effect on events at small q^2.

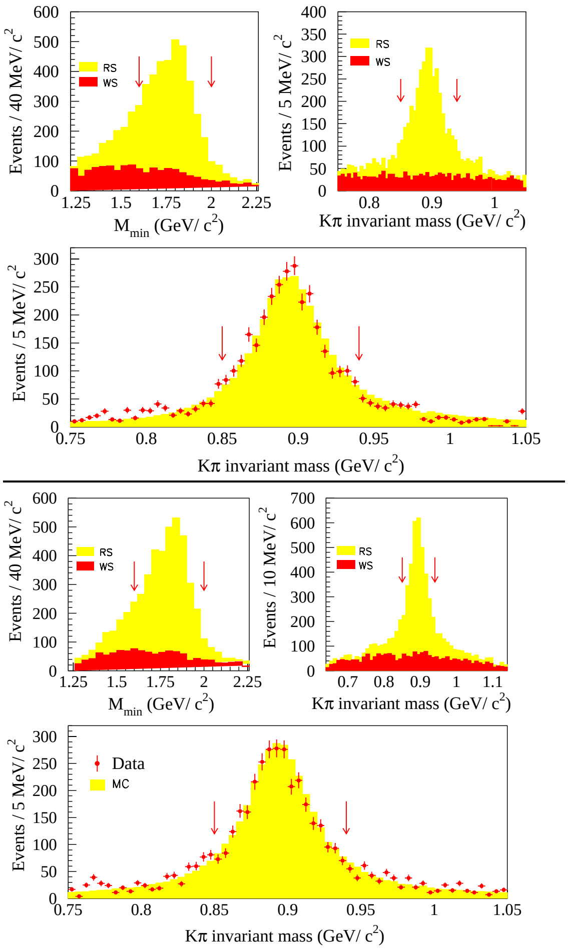

Figure 3: E791 signals for $D^+ \to \overline{K}^{*0}\ell^+\nu$. Histograms of M_{min} and $K\pi$ invariant mass for the electron (top) and muon (bottom) modes after all other event selection cuts have been applied. In the top row of figures for each mode, right-sign candidates (RS, light shading) are $K^-\pi^+\ell^+$; wrong-sign events (WS, dark shading) are $K^+\pi^-\ell^+$. In the bottom plots for each mode, subtracted data (RS-WS, data points) are compared with Monte Carlo simulation (shaded histograms).

The top plot in figure 5 compares the existing $r_2(0)$ and $r_V(0)$ measurements; all are consistent within errors. The E791 data now dominate the unofficial world average of

$$r_2(0) = 0.73 \pm 0.07, \quad r_V(0) = 1.85 \pm 0.09.$$

The bottom plot in figure 5 compares the E791 measurement to several theoretical predictions (listed in reference [?]) from HQET and quark models (no error flags), and from lattice QCD (with error flags) calculations. With the excep-

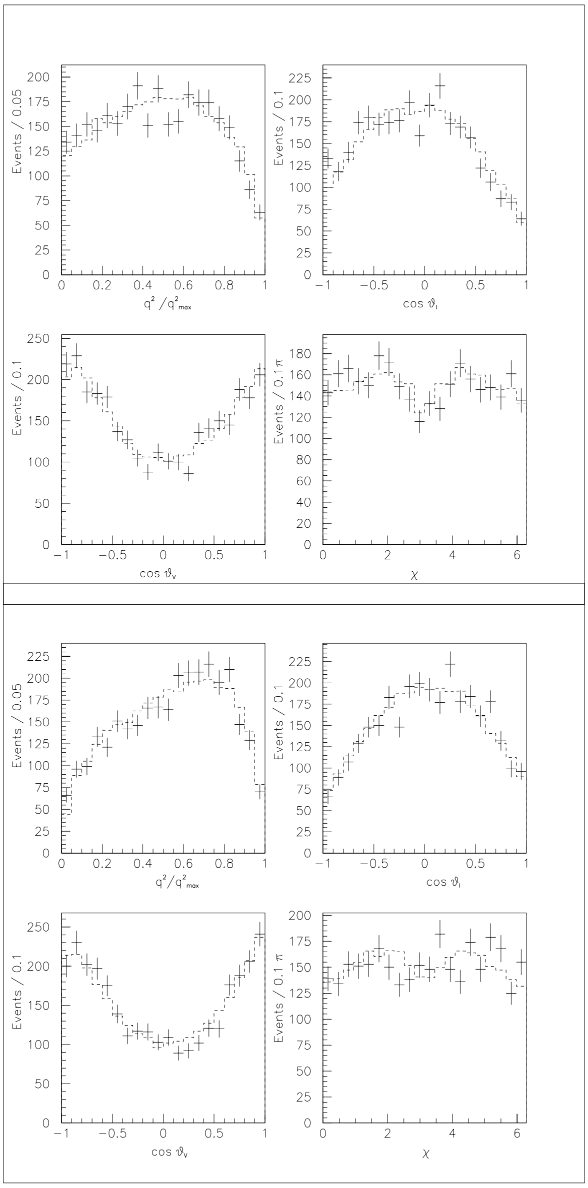

Figure 4: From E791 $D^+ \to \overline{K}^{*0}\ell^+\nu$: comparison of integrated distributions of data (points) in the decay variables q^2/q^2_{max}, $\cos\theta_l$, $\cos\theta_V$ and χ with Monte Carlo simulation using best-fit values for the form factor ratios (dashed histograms). Note that electron (top) and muon (bottom) decay variable distributions are very similar except for q^2/q^2_{max}, in which muon mass effects are evident at small q^2.

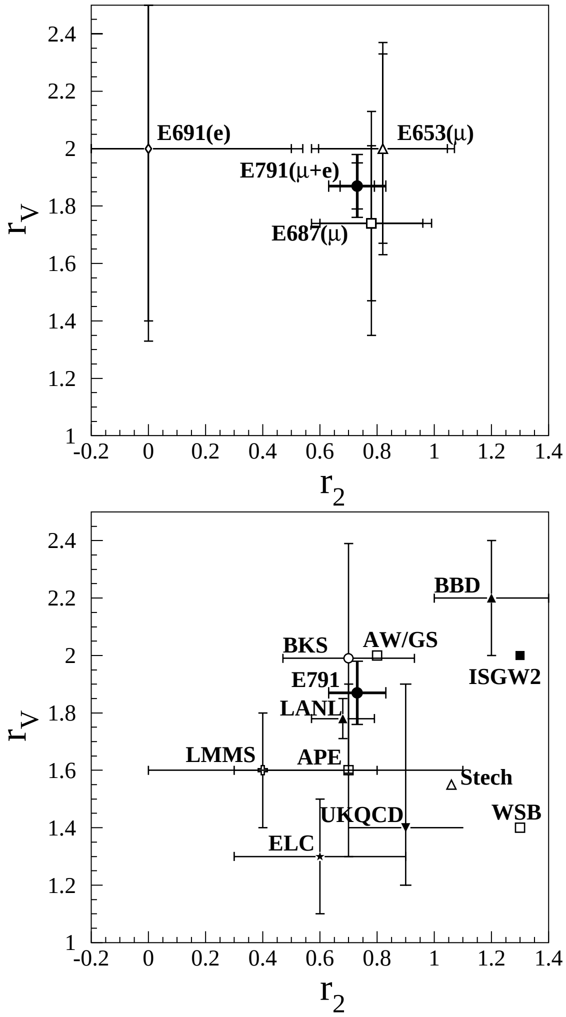

Figure 5: Top: Comparison of experimental measurements of form factor ratios r_V and r_2 for $D^+ \to \overline{K}^{*0}\ell^+\nu$ in the muon (μ), electron (e) and combined ($\mu + e$) channels. The smaller error bars indicate the statistical errors and the larger ones indicate the statistical and systematic errors added in quadrature. Bottom: Comparison of theoretical predictions with the E791 ($\mu + e$) result; the theory references are given in reference [**?**]. It is evident that E791's small errors present a challenge to theorists.

tion of the excellent LQCD result from LANL [**?**], the data error bars are now considerably smaller than those of the calculations.

All measurements so far have constrained the q^2 dependence of the form factors to the dipole form and pole masses described above, a plausible but untested assumption. To first order, the dipole form reduces to

$$f(q^2) \approx f(0)\cdot(1+q^2/M^2_{A,V}) = f(0)\cdot(1+\rho^2_{A,V}q^2).$$

E791 attempted to measure the q^2 dependence with 3,000 electron events by including a form factor slope parameter ρ^2 in addition to $r_2(0)$ and

$r_V(0)$; the assumed form factor q^2 dependence was $(1+\rho^2_{A,V}q^2)$, with the ratio of ρ^2_A/ρ^2_V constrained to the usual M^2_V/M^2_A. Strong correlations make this a tricky business; for example, r_2 dominates at low q^2, r_V at high q^2. E791 found that $r_V(0)$ was the same with free or constrained slope, but that $r_2(0)$ and ρ^2_A were strongly anticorrelated, with r_2 rising by two standard deviations when the q^2 slope was unconstrained. Their slope result was

$$\rho^2_A = -0.06 \pm 0.10 \text{ GeV}^2,$$

barely consistent with the expected

$$\rho^2_A = 1/M^2_A = +0.16 \text{ GeV}^2.$$

These effects clearly must be understood when higher statistics data are available. FNAL E831 (FOCUS) has shown [?] K^* signals from this mode, based on 23% of their data, that extrapolate to about 60,000 events in the combined e, μ channels, an order of magnitude larger than the E791 sample. This should be enough to nail the q^2 dependence.

2.2 $D^+_s \to \phi\ell^+\nu$ and SU(3)

If SU(3) symmetry is approximately valid, replacing a spectator $\bar{d}$ quark by a spectator $\bar{s}$ quark in the decaying charm meson should have little effect on the form factors. With the recent results from E791 in both the $D^+ \to \overline{K}^{*0}\ell^+\nu$ and $D^+_s \to \phi\ell^+\nu$ channels, this hypothesis has been tested experimentally.

The E791 signals for $D^+_s \to \phi\ell^+\nu$ [?] are shown in figure 6. There are again two signatures, a ϕ peak in K^+K^-, and a cusp at the D_s mass in M_{min}. For this two-kaon decay, RS events are defined as those with same lepton and parent charge, WS as opposite lepton, parent charges. In this case, one expects 2·WS to measure combinatoric background. In the electron channel, E791 has 166 RS, 11 WS, and 144 net signal events, and finds

$$r_2(0) = 1.64 \pm 0.34 \pm 0.20,$$

$$r_V(0) = 2.24 \pm 0.47 \pm 0.21;$$

in the μ channel, they have 161 RS, 17 WS, and 127 net signal events, and find consistent

$$r_2(0) = 1.57 \pm 0.25 \pm 0.19,$$

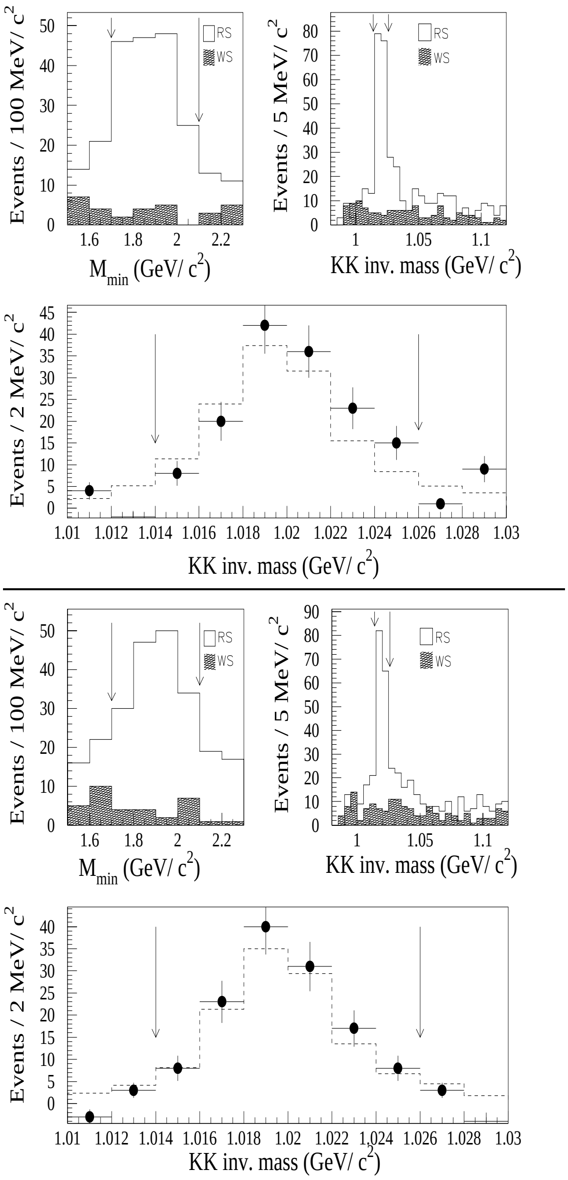

Figure 6: E791 signals for $D^+_s \to \phi\ell^+\nu$. Histograms of M_{min} and K^+K^- invariant mass for the electron (top) and muon (bottom) modes after all other event selection cuts have been applied. In the top row for each mode, right-sign candidates (RS, unshaded) are $K^+K^-\ell^+$; wrong-sign events (WS, shaded) are $K^+K^+\ell^-$. In the bottom row for each mode, subtracted data (RS-2×WS, data points) are compared with a Monte Carlo simulation (dashed histograms). Arrows indicate the mass ranges for the final event sample.

$$r_V(0) = 2.31 \pm 0.54 \pm 0.26.$$

The combined result is

$$r_2(0) = 1.57 \pm 0.25 \pm 0.19,$$

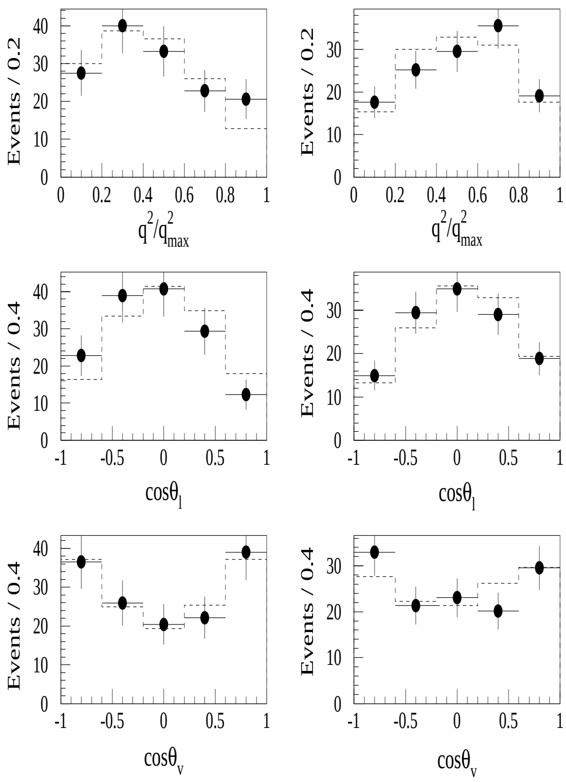

Figure 7: From E791 $D_s^+ \to \phi\ell^+\nu$; electron mode (left), and muon mode (right). Comparison of integrated distributions of data (points) in the decay variables q^2/q^2_{max}, $\cos\theta_l$ and $\cos\theta_V$ with Monte Carlo simulation using best-fit values for the form factor ratios (dashed histograms).

$$r_V(0) = 2.27 \pm 0.35 \pm 0.22.$$

Single-variable distributions are compared with Monte Carlo in figure 7.

Experimental form factor results for $D_s^+ \to \phi\ell^+\nu$ are compared in the top plot of figure 8. Again, all measurements are consistent. The unofficial world average is $r_2(0) = 1.58 \pm 0.25$, $r_V = 1.97 \pm 0.32$. In the bottom plot in figure 8, the E791 experimental result is compared to theoretical predictions (listed in reference [?]), and also to the E791 measurements for $D^+ \to \overline{K}^{*0}\ell^+\nu$ to test the SU(3) hypothesis.

The two experimental results for r_V agree well with each other and with calculations. However, the E791 r_2 for D_s^+ is more than two standard deviations higher than the E791 r_2 for D^+. The experimental discrepancy is even larger – about 3.3 standard deviations – when world averages are used instead, implying a significant deviation from SU(3) in r_2. It appears that SU(3) symmetry holds less well for r_2 than for r_V, for reasons not yet understood.

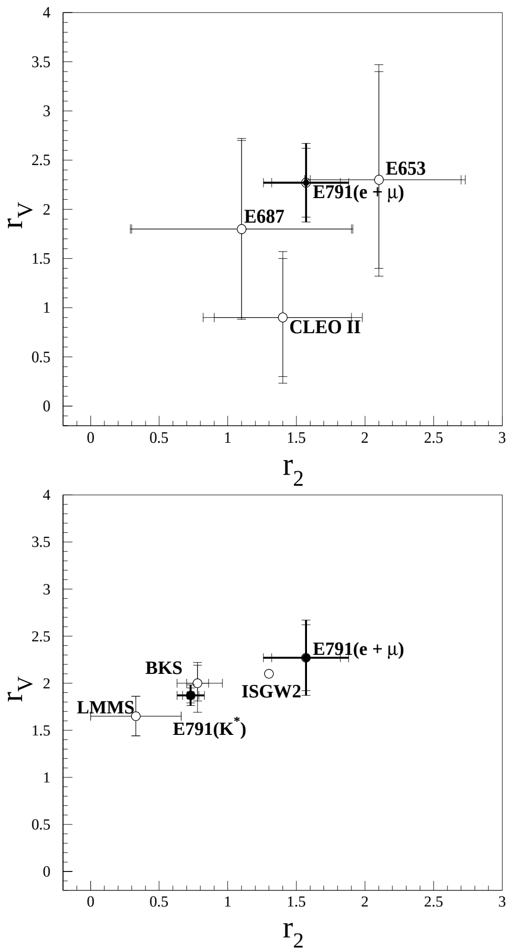

Figure 8: Top: Comparison of experimental measurements of form factor ratios r_V and r_2 for $D_s^+ \to \phi\,\ell^+\nu$. The smaller error bars indicate the statistical errors and the larger ones show the statistical and systematic errors added in quadrature. Bottom: Comparison of E791 measured form factor ratios for $D_s^+ \to \phi\,\ell^+\,\nu$ with theoretical predictions for $D_s^+ \to \phi\,\ell^+\,\nu$ and with E791 measurements for $D^+ \to \overline{K}^{*0}\,\ell^+\,\nu$. Theoretical references are listed in reference [?].

The spread in theoretical predictions for r_2 is also larger than the spread of predictions for r_V. This happens because the quoted theoretical papers predict r_2 equal to 10% for the two decays (i.e., they obey SU(3) symmetry). It is thus difficult for them to agree with both D^+ and D_s

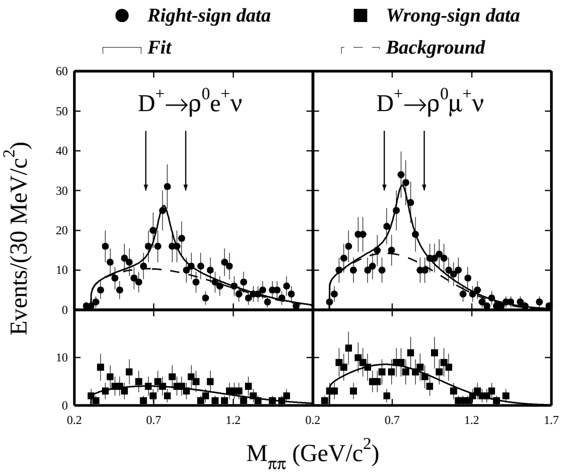

Figure 9: Signals for $D^+ \to \rho^0 \ell^+ \nu$ from FNAL E791, showing that getting a signal clean enough for form factor physics will not be easy. Both kinematic and Cherenkov cuts have been used to obtain this sample. Plotted is invariant mass of $\pi\pi$ for RS and WS; combinatoric background should be $\approx$ 2·WS. Left: electron mode; right: muon mode.

measurements.

FNAL E831 (FOCUS) expects about 10,000 events in the $D_s^+ \to \phi \ell^+ \nu$ channel. They will certainly obtain precision measurements of form factor ratios with assumed q^2 dependence, and could possibly make a rough measurement of q^2 dependence.

3. Prospects for $D^+ \to \rho^0 \ell^+ \nu$

Like the CS decay to $\pi \ell^+ \nu$ discussed earlier, this CS decay to a vector daughter is interesting because of its connection to measuring $|V_{bu}|$. It is even more difficult, because background must be reduced to the point where the decay analyzers in figure 2 can be used reliably. No form factor measurements exist.

Figure 9 shows the present state of the art, from E791 [?] in which the branching ratio from this mode was determined; FNAL E687 [?] has a signal of similar quality in the muon mode. The plots shown are $\pi\pi$ invariant mass for the e and μ modes; the net ρ^0 signals are 74 ± 15 and 81 ± 17 events, respectively. RS and WS events are defined as for the ϕ case above, and combinatoric background should be $\approx$ 2·WS.

As in the CS pion mode, Cherenkov identification is not nearly enough to reject feedthrough from CF decays. Several kinematic tricks were used to obtain the signal in figure 9: for example, the M_{min} distributions with pion and kaon hypotheses have only $\sim$ 50% overlap, allowing cuts which enrich the CS mode at the expense of statistics, and at the expense of biasing the decay-analyzing variables.

FNAL E831 (FOCUS) should have about 2,000 events in this mode. With this large sample, they should be able to trade statistics for a cleaner signal with creative event-selection criteria. But getting form factors, even with assumed q^2 dependence, will be a heroic analysis effort.