# Supersymmetry Breaking in M-theory

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ABSTRACT: We present a way in which two higher dimensional, spatially separated gauge sectors are combined to form a consistent four dimesnional supergravity. We reconsider the issue of the low energy supersymmetry breaking in the field theory limit of the strongly coupled heterotic string. We derive general observable soft supersymmetry breaking operators for nonstandard perturbative embeddings. We discuss racetrack scenarios in M-Theory.

## 1. Introduction

In their analysis of the effective field theory limit of the strongly-coupled heterotic $E_8 \times E_8$ string theory, Horava and Witten [1, 2] constructed a consistent eleven-dimensional supergravity on a manifold $M_4 \times X \times S^1/Z_2$, coupled to ten-dimensional Yang-Mills models on the fixed hyperplanes of the $S^1/Z_2$ orbifold. Witten [1] also solved the equations of motion along the eleventh dimension on the orbifold $S^1/Z_2$, and found the correct six-dimensional compactification that preserves four unbroken supercharges in the presence of non-trivial background components of the antisymmetric tensor field strength $G_{ABCD}$. He also calculated the gravitational constant in four dimensions and the gauge couplings on both the visible and hidden walls.

On the basis of general arguments, this theory should reduce to some four-dimensional $N = 1$ supergravity theory in the infrared limit. Knowing that the final effective theory is four-dimensional $N = 1$ supergravity, one way to obtain the complete Lagrangian is simply to read the Kähler potential, superpotential and gauge kinetic functions off from the direct $d = 11 \to 4$ reduction of the relevant eleven-dimensional terms. With the full four-dimensional Lagrangian at hand, one can study the properties of its vacuum, such as supersymmetry breaking, and thereby understand

the infrared limit of the strongly-coupled heterotic superstring.

We follow this route in the first part of this note, and contrast the generic features of supersymmetry breaking due to gaugino condensation in the strongly-coupled case [3] with the better-known weakly-coupled case [4]. An important difference is that supersymmetry is generically broken by nonzero expectation values of both $F^S$ and $F^T$ in the strongly-coupled string. The effective potential of the four-dimensional supergravity in the strongly-coupled case cannot be written simply as a perfect square, complicating the minimization problem, which is not solved simply by minimizing the $|F^S|^2$ term alone. This observation may be welcome, given the phenomenological interest [5] in the dilaton-dominated scenario for supersymmetry breaking. However, we also demonstrate that the potential does not vanish generically.

The above approach is appropriate if the condensation scale $\Lambda$ is smaller than the threshold $m_5 = 1/R_5$ for five-dimensional Kaluza-Klein excitations. However, if $\Lambda > m_5$, in order to understand the pattern of supersymmetry breaking, one needs the full four-dimensional Lagrangian constructed by the compactification chain $d = 11 \to 5 \to 4$ [6, 7, 8, 9, 10, 11]. For this, one must first study how the low-energy four-dimensional model arises out of the two $d = 10 \to 4$ sectors which are spatially disconnected in the original $d = 11 \to 5$ theory, in other words, how

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the two gauge sectors are combined together in course of the compactification process. The way this happens is outlined in the second part of the note.

The complete compactification of that five dimensional model down to four dimensions has not been performed yet. An important aspect of the attempt at the establishing of the detailed relation between the Horava-Witten model and the 4d supergravity is that the Horava-Witten Lagrangian, although anomaly free and supersymmetric, can be trusted in its role of the approximation to the strongly coupled string to a limited extent. It contains terms which are of the order $\kappa^{2/3}$ with respect to the gravitational action plus these terms of the order $\kappa^{4/3}$ which are necessary to retain supersymmetry in 11d.

The question of supersymmetry transmission through the bulk in the five-dimensional supergravity was discussed in previous papers [9], [10], [7], [12], [13], [14], [15], [16]. The reader should be warned that different approaches seem to give somewhat different results.

## 2. Supersymmetry Breaking in Four Dimensions

We begin by following the first route described in the Introduction. We recall that one obtains from the Horava-Witten model the Kähler functions and the gauge kinetic functions for the standard and non-standard embeddings, consistently to order $\kappa^{2/3}$, that is with the threshold corrections to the gauge kinetic functions included. This is sufficient to reconstruct directly the parts of the scalar potential that are relevant for seeing the supersymmetry-breaking structure in the effective four-dimensional supergravity theory arising from the strongly-coupled heterotic string, *if* gauginos condense at low energies, below the scale $m_5 = \frac{1}{R_5}$. In this case, it is fully adequate to work entirely within the four-dimensional supergravity framework, as we assume in this Section. However, if gauginos condense at higher energies, above the scale $m_5 = \frac{1}{R_5}$, the full five-dimensional approach of the following Section is required.

We first recall the way the vev of gaugino bilinears $\langle \lambda^a \lambda^b \rangle$ enters the effective scalar poten-

tial of the four-dimensional supergravity [17] [1]. Using the canonical normalization in four dimensions for the gravitational, gauge and gaugino kinetic terms, the relevant part of the Lagrangian is

$$V = e^K g^{i\bar{j}}(D_i W + \tfrac{1}{4}e^{-K/2}\partial_i f_{ab}\langle\lambda^a\lambda^b\rangle)$$
$$(D_{\bar{j}}W + \tfrac{1}{4}e^{-K/2}\partial_{\bar{j}}\bar{f}_{ab}\langle\bar{\lambda}^a\bar{\lambda}^b\rangle) + ..., \quad (2.1)$$

where $g^{i\bar{j}}$ is the inverse Kähler metric and rest of the notation is standard [17].

Comparing (2.1) with the well-known general expression $V = g_{i\bar{j}}F^i F^{\bar{j}} - 3e^G$ for the four-dimensional potential in terms of the $F^i$, the auxiliary fields for the chiral multiplets, we read off the modified expressions for the auxiliary fields in the presence of the condensates

$$F^i = e^{K/2}g^{i\bar{j}}(D_{\bar{j}}\bar{W}+\frac{1}{4}e^{-K/2}\partial_{\bar{j}}\bar{f}_{ab}\langle\bar{\lambda}^a\bar{\lambda}^b\rangle) \quad (2.2)$$

In the following, we match this expression explicitly to the fermionic bilinears in the effective Lagrangian, since [3] this is a better description when the gauge kinetic function depends on more than one modulus, and the gaugino bilinears are among the terms which we obtain directly from the Calabi-Yau reduction.

In the weakly-coupled heterotic string, at tree level, the gauge kinetic function is universal: $f = S$, and the Kähler function for the illustrative case of a single universal modulus $T$ is $K = -\log(S + \bar{S}) - 3\log(T + \bar{T})$. In this case, the full scalar potential reduces to

$$V = g_{S\bar{S}}|F^S|^2, \quad (2.3)$$

since $g_{T\bar{T}}|F^T|^2 = 3|W|^2 e^K$. This relation is equivalent to the vanishing of the perfect square containing the gaugino condensates in the ten-dimensional effective action of the weakly-coupled heterotic superstring [18]. In this way, we find

$$F^S = 0, \quad F^T \neq 0 \quad (2.4)$$

In general models there are additional contributions to the effective potential which depend on the modulus $T$, however, in all the models studied so far, the vacuum relation $F^S \ll F^T$ persists, and supersymmetry breaking occurs along

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[1]We denote by $\lambda^a$ the gaugino components, where $a$ is an adjoint group index, and $i$ labels complex moduli fields.

the $T$ direction in the moduli space. The technical reason is that the dependence on $S$ factorizes out in simple modular-invariant models of condensation, and the equations of motion, i.e., dynamics, tell us that $F^S$ is small. The vev of the modulus $T$ is rather small in these models, close to unity in supergravity units.

The structure of the supersymmetry-breaking sector is significantly modified n the strongly-coupled regime. The Kähler function for the universal moduli $S$, $T$ is the same, but the classical, or tree level, gauge kinetic functions are changed, and are different for different walls: $f_{1,2} = S \pm \xi_0 T$ where

$$\xi_0 = -\frac{\pi \rho_0}{2(4\pi)^{4/3}\kappa^{2/3}} \frac{1}{8\pi^2} \int_X \omega_K \wedge (\, trF^{(1)} \wedge F^{(1)}$$
$$-\frac{1}{2} trR \wedge R\,). \qquad (2.5)$$

Here $\omega_K$ is the Kähler $(1,1)$-form, and the topological integral over Calabi-Yau space can be parametrized in terms of gauge and gravitational instanton numbers characterizing the embedding [19]: $\xi_0 = \frac{n_{F_1} - \frac{1}{2}n_R}{32\pi^3}$. The interesting region of moduli space is where $S = \mathcal{O}(2)$ and $T = \mathcal{O}(80)$ [19, 20]. Hence, we are not interested in mechanisms which generate minima of the potential at $T \approx 1$, but need some new mechanism which generates a minimum in the region of current interest. We do not discuss any specific mechanism here, but just state the possibilities opened up by the current form of the kinetic functions.

First, we note that $S$ and $T$ enter the kinetic functions, and hence any nonperturbative potential, in quite a symmetric way. The relative coefficient $\xi_0$ which weights the contribution of $T$ changes from model to model. In the elliptic-fibration models of [20], this number is smaller than 0.025. It could in principle be either much larger or much smaller in more general constructions. Because of this greater symmetry between $S$ and $T$, there is no obvious reason why $F^S$ should be much smaller than $F^T$ in the generic case, in the interesting portion of moduli space. In the strongly-coupled case, we obtain

$$V = e^K (S + \bar{S})^2 \left| -\frac{W}{S+S} + \frac{1}{4} e^{-K/2}(\Lambda_1^3 + \Lambda_2^3) \right|^2$$
$$+ e^K \frac{(T+\bar{T})^2}{3} \left| -\frac{3W}{T+T} + \frac{1}{4}\xi_0 e^{-K/2}(\Lambda_1^3 - \Lambda_2^3) \right|^2$$
$$- 3e^K |W|^2. \qquad (2.6)$$

and the result for the $F$ terms is:

$$F^S = \frac{1}{4}(S + \bar{S})^2 (\Lambda_1^3 + \Lambda_2^3) \qquad (2.7)$$

$$F^T = \frac{1}{12}(T + \bar{T})^2 \xi_0 (\Lambda_1^3 - \Lambda_2^3) \qquad (2.8)$$

It is clear that supersymmetry is unbroken: $F^S = F^T = 0$ if and only if both condensates vanish. Even if there is only one condensate, both $F^S$ and $F^T$ are nonzero. Moreover, if condensates on both walls are switched on simultaneously, no matter in what proportion, supersymmetry is always broken in four dimensions. In particular, even when the two condensates are switched on with the same magnitude, and opposite signs, supersymmetry is formally broken [2], contrary to [21]. Let us note, that we suppress here the possibility of a constant superpotential contribution, which could arise as a vev of the gauge and/or gravitational Chern-Simons forms on either wall. The inclusion of these terms, the details of which are beyond the scope of the present note, does not change the general picture unless one considers very special situations which are unlikely to arise dynamically.

A further consequence of $F^S \neq 0$ is a nonzero scalar mass, which arises from (2.1) upon substituting the correction $-\delta K_S = \pm \xi_0 |A^q|^2/(S + \bar{S})^2$ [3], which gives soft scalar masses proportional to $\pm \xi_0 F^S$.

Finally, we examine the ratio of the two $F$ terms

$$\frac{F^S}{F^T} = \frac{3}{\xi_0} \frac{\Lambda_1^3 + \Lambda_2^3}{\Lambda_1^3 - \Lambda_2^3} \left(\frac{S}{T}\right)^2. \qquad (2.9)$$

In the present region of moduli space, the ratio of $S/T$ is of the order $1/40$ or so, so it would not require very much fine-tuning to arrange the magnitudes of the condensates in such a way that the ratio $F^S/F^T$ is of the order of unity or larger. To make the possibility of the mixed $S, T$-moduli-driven scenario more plausible, we look at the ratio $F^S/F^T$ more carefully. As pointed

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[2]The magnitude of the breaking is to be determined from the vacuum solution of the effective potential, but we would expect $m_{3/2} = \mathcal{O}(\Lambda^3/M_{Pl}^2)$.

[3]This correction arises from the correction to the metric of the Calabi-Yau space, i.e., to the factor of $\sqrt{g_{(6)}}$ which multiplies the kinetic terms of the four-dimensional charged scalars [22].

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out in [19], one can easily express the expectation value of $T$ through the observable quantities $T = (\frac{M_P}{M_{GUT}})^2 \frac{\alpha_{GUT}^{1/3}}{2^{17/3}\pi^3}$. Then we can express $S$ as $S = \frac{1}{4\pi\alpha_{GUT}} - \xi_0 T$. As a result, we obtain the ratio of the $F$ terms as a function of $\xi_0$:

$$\frac{F^S}{F^T} = \frac{3}{\xi_0} \left( \frac{2^{11/3}\pi^2 M_{GUT}^2}{\alpha_{GUT}^{4/3} M_P^2} - \xi_0 \right)^2 \frac{\Lambda_1^3 + \Lambda_2^3}{\Lambda_1^3 - \Lambda_2^3}. \tag{2.10}$$

The prefactor multiplying the condensates can be studied as a function of $\xi_0$, when we fix the observables at their MSSM values. One finds that the prefactor vanishes at $\bar{\xi}_0 \approx 0.025$, but grows quickly to values of $\mathcal{O}(1)$ for larger $\xi_0$, and to the values $\geq 0.07$ at $\xi_0 \leq 0.01$. For negative $\xi_0$, i.e. in the regime of 'strong' unification, the value of the prefactor is always larger than $1/10$. Thus, it is possible to obtain quite a large value of $F^S$, and even the extreme option of $S$-dilaton-driven supersymmetry breaking cannot be completely excluded in the strongly-coupled heterotic string. This could have interesting consequences, given the promising results of phenomenological investigations of this limit in the weakly-coupled string.

We finish this section with the list of the soft terms found here in the four-dimensional supergravity approach. We generalize the earlier results [13, 14, 23, 24, 25] by considering non-standard embeddings in which charged matter is present on both walls, i.e., in both gauge sectors, and we allow for condensates to form on both walls [4]. Restoring powers of the reduced Planck mass $M$, the physical gravitino mass in the case of a vanishing cosmological constant in four dimensions is:

$$m_{3/2}^2 = \frac{(S+\bar{S})^{2/3}}{2^{2/3}12M^4}(\Lambda_1^3 + \Lambda_2^3)^2 + \frac{(T+\bar{T})^2}{432M^4}\xi_0^2(\Lambda_1^3 - \Lambda_2^3)^2 \tag{2.11}$$

and the mixing angle $\theta$ introduced through the relation $\frac{F^S}{F^T} = \sqrt{3}\frac{S+\bar{S}}{T+\bar{T}}\tan\theta$ is given by

$$\tan\theta = \sqrt{3}\xi_0^{-1}\frac{S+\bar{S}}{T+\bar{T}}\frac{\Lambda_1^3 + \Lambda_2^3}{\Lambda_1^3 - \Lambda_2^3}. \tag{2.12}$$

Assuming that the CP-violating phases vanish, we obtains trilinear scalar terms of the form

$$A = \sqrt{3}m_{3/2}\left(\sin\theta\left(-1 \pm \xi_0\frac{3(T+\bar{T})}{3(S+\bar{S})\pm\xi_0(T+\bar{T})}\right)\right.$$

$$\left. + \sqrt{3}\cos\theta\left(-1 + \frac{3(T+\bar{T})}{3(S+\bar{S})\pm\xi_0(T+\bar{T})}\right)\right) \tag{2.13}$$

and gaugino masses

$$M_{1/2} = \frac{\sqrt{3}m_{3/2}}{(S+\bar{S})\pm\xi_0(T+\bar{T})}\left(\sin\theta(S+\bar{S})\right.$$

$$\left. \pm \xi_0\cos\theta\frac{(T+\bar{T})}{\sqrt{3}}\right). \tag{2.14}$$

Note that there is a difference of sign between the expressions linear in $\xi_0$ corresponding to different walls. This can have consequences in some of nonstandard embedding models, where, e.g., matter with Standard Model hypercharge may exist on both walls. The dilaton-dominated limit corresponds to $\sin\theta \to 1$. We can see from the formulae for the $A$ terms and gaugino masses that even in this limit there is non-universality between terms containing charged fields from different walls, see [26] for details.

Using standard supergravity formulae one can easily write down the formulae for the soft scalar masses, assuming some reasonable form of the corresponding Kähler potential, and they are given in [26], [27]. One again notices the characteristic changes of sign of some terms when one goes from one wall to the other, and the universality of these terms is violated even in the dilaton-driven supersymmetry-breaking limit.

It is useful to generalize the above analysis to the case of an arbitrary number of condensates. Hence, following the reference [26] we shall summarize briefly the main features of the racetrack models with condensates forming in two different gauge sectors, with different gauge kinetic functions $f_{1,2} = S \pm \xi_0 T$ [5].

First, let us look for the flat space supersymmetric points of the effective potential. One reason is purely technical, namely it is much easier to find such candidate points than to look for broken supersymmetry solutions to the full equations of motion. Secondly, as we expect the realistic supersymmetry breaking scale to be hierarchically smaller than the Planck scale, one can expect the relevant points where supersymmetry is only slightly broken, to be located near the globally supersymmetric points (although the existence of remote relevant susy breaking points cannot be excluded in general). As we are looking for flat space solutions, we shall assume the

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[4]However, we do not consider five-branes in the bulk.

[5]See also discussion in [28].

vacuum expectation value of the total superpotential to be zero, which is consistent in the picture with explicit gaugino bilinears, as it does not lock to each other the values of various condensates. Then, the scalar potential takes the form

$$V = \frac{(S + \bar{S})^2}{16M^2}|\sum_i \Lambda_i^3|^2 + \xi_0^2 \frac{(T + \bar{T})^2}{48M^2}|\sum_i \epsilon_i \Lambda_i^3|^2$$

(2.15)

(where $\epsilon_i = \pm 1$ and the rest of the notation is standard and given in [27]), and is equivalent to the potential obtained in the globally supersymmetric limit in the effective superpotential approach to multiple condensates. It is easy to see that existence of supersymmetric minima implies that the sum of condensates vanishes separately on each wall

$$\sum \Lambda_{i+}^3 = 0 = \sum \Lambda_{i-}^3 \qquad (2.16)$$

at such points in field space. Here $i+, i-$ run over the number of condensates present on wall 1 and wall 2 respectively and $\{i\} = \{i+, i-\}$.

In the case of at least two condensates on each wall, equations (2.16) are two independent equations for two complex variables $e^{S \pm \xi_0 T}$, so generically they have a solution at finite values of $S$ and $T$. The situation is such, that condensates on each wall optimize themselves and supersymmetry is unbroken, but the values of $S$ and $T$ become fixed. This is an interesting possibility, and it would be an interesting exercise to check which values of $Re(S)$ and $Re(T)$ can be obtained in such a setup, but in this paper we want to stay specifically within the class of calculable perturbative nonstandard embeddings discussed in [20], and there doesn't seem to be enough space for $\geq 4$ condensates with realistically low condensation scales.

Hence we have to look at the vacua with three or less condensates. These are the interesting cases, given our comments above. First, when we want to have 3 condensates arranged on different walls, then one of them must be on one wall, and two on the opposite. Then it follows immediately that to fulfill the unbroken susy conditions the single condensate would have to vanish. With three nonvanishing condensates on both walls we therefore always have broken supersymmetry. The same applies, as pointed out

in [27], to the case of two condensates on different walls.

When we have several condensates on a single wall, then this works similarly to the weakly coupled case: supersymmetry is unbroken, but only $S + \xi_0 T$ or $S - \xi_0 T$ is fixed. To have a hope to fix both moduli within the simple, and perhaps most appealing, version of the racetrack scheme one clearly needs to consider condensates on both walls.

To conclude, in the most interesting case of two or three condensates on different walls, we can exclude the existence of flat space, unbroken supersymmetric ground states. This is not so bad however, because the existence of remote minima, disconnected from any globally supersymmetric state, cannot be excluded by a general reasoning. In fact, if one finds any proper minimum of the effective potential in these cases, supersymmetry is guaranteed to be broken there. However, to have a vanishing cosmological constant at such a minimum, one has to invoke a nonzero expectation value of the superpotential, and it would have to be of the order of $\Lambda_{eff}^3$ if we are to get the usual hierarchy $F \approx \Lambda_{eff}^3/M$. One source of such effectively constant terms in the superpotential could be condensates of the Chern-Simons forms, but then the question of the scale and 'stiffness' of these condensates arises, which we shall not discuss here. It is worth noting that even if the constant in the superpotential is present, then on the basis of the dynamics described by the 4d effective potential one cannot tell whether it makes $F_T$ or $F_S$ vanish. In fact, the most plausible situation in such a case is that supersymmetry is unbroken, moduli fixed, but the cosmological constant is nonzero.

## 3. The Five-Dimensional Connection between Four-Dimensional Worlds

In this Section, we construct explicitly the four-dimensional supergravity Lagrangian that arises from the sequence of compactifications: $11 \rightarrow 5 \rightarrow 4$, remembering that the result can be trusted only in the lowest order in $\kappa^{2/3}$. As already mentioned, this approach is inescapable *if* gauginos condense at high energies, above $m_5 = \frac{1}{R_5}$. In this case, the four-dimensional Lagrangian is

born from two spatially separated gauge sectors. As will be clear from our construction, the result should be a four-dimensional supergravity theory in the approximation $\xi_0 = 0$.

Since, in this Section, we first construct the five-dimensional theory, we perform the Weyl rotation of the metric which gives the canonical Einstein-Hilbert action in five dimensions. The relation between the canonical eleven- and five-dimensional metrics is $g_{MN}^{(11)} = (e^{-2\sigma} g_{\mu\nu}^{(5)} ; e^{\sigma} g_{ab}^{(0)})$. In this notation, we take $e^{3\sigma} = Re(S)$, where $S$ is the $Z_2$-even scalar from the universal hypermultiplet [9, 10].

In what follows, we shall use the same symbols for both five-dimensional moduli, e.g., $\sigma$, and for the corresponding four-dimensional quantum fields, whenever it is obvious from the context which ones we actually have in mind.

If we restrict ourselves to a Calabi-Yau space with $h_{(2,1)} = 0$, then the decomposition of gauge fields with compact indices which defines matter fields in **27**s of $E_6$ is $A_i = A^{kp}(x)\omega_i^{kj}(y)T_{jp}$, where the $\omega_i^{kj}(y)$ are harmonic $(1,1)$-forms, and the $T_{jp}$ are generators of $E_6$. We note the following properties of the generators: $\mathrm{Tr}T_{ip}T_{jq}T_{kr} = \epsilon_{ijk}d_{pqr}$, $\mathrm{Tr}T_{jp}T_{j'p'} = \delta_{jj'}\delta_{pp'}$, and the appropriate expansions for the field strength: $F_{\mu a} \rightarrow \partial_\mu C^{Kp}\omega_a^{Kj}T_{jp}$, $F_{\nu b} \rightarrow \partial_\nu C^{Lp'}\omega_b^{Lj'}T_{jp'}$. Finally, we shall use the following decomposition of the regular part of $G_{abc11}$ in terms of massless Calabi-Yau modes [10]

$$(G_{11})_{abc} = 2\partial_{11}C_0\overline{\Omega}_{abc} + h.c. \qquad (3.1)$$

With these conventions and definitions, the result of the reduction of the 'perfect square', given in [2], down to five dimensions is

$$\mathcal{L}_{sq} = -\frac{V_0}{12\kappa^2}\int d^5x \sqrt{g_{(5)}}e^{-3\sigma}g^{55}\left(2\partial_5 C_0 \right.$$
$$+ 4\frac{\kappa^2}{\sqrt{2}\lambda^2}\delta^{(m)}P^{(m)}$$
$$\left. - \frac{1}{32\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}e^{\frac{9}{2}\sigma}(g_{55})^{3/4}\delta^{(m)}\bar{\epsilon}_m^{(4)}\epsilon_m^{(4)}\right)^2 (3.2)$$

where $P^{(m)}(A) = \lambda_{KLM}d_{pqr}A^{Kp}A^{Lq}A^{Mr}$, $m = 1, 2$ labels the walls, and we have used the canonical normalization for gauginos in four dimensions. Here $e^{\sigma(x,x^5)}$ is the five-dimensional variable measuring the volume of the Calabi-Yau space along the orbifold interval, in units of the fiducial volume $V_0$. This is the real part of the $Z_2$-even scalar from the universal hypermultiplet:

$Re(S) = e^{3\sigma(x^5)}$. The relation between $S$ and the four-dimensional fields $\tilde{S}$, $\tilde{T}$ is

$$e^{3\sigma(x^5)} = e^{3\tilde{\sigma}} + \xi_0 e^{\tilde{\gamma}}(1 - \frac{2x^5}{\pi\rho_0}). \qquad (3.3)$$

In the above expression, $e^{\tilde{\sigma}} = Re(\tilde{S})$, and $e^{\tilde{\gamma}} = Re(\tilde{T}) = \sqrt{g_{55}(x^\mu)}$.

To construct the effective theory in four dimensions, one has to integrate out the components of the five-dimensional fields which do not correspond to massless degrees of freedom in four dimensions, and the natural way to do this is through the solution of the equations of motion along the dimension which one wishes to compactify. To the lowest order in $\kappa^{2/3}$, i.e., to the zeroth order in $\xi_0$, the equation is

$$\partial_5^2 C_0 = \sum_m \left(-\frac{\sqrt{2}\kappa^2}{\lambda^2}P^{(m)}\right.$$
$$+ \frac{1}{64\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}e^{\frac{9}{2}\sigma}(g_{55})^{3/4}\bar{\epsilon}_m^{(4)}\epsilon_m^{(4)}\right)\partial_5\delta^{(m)}$$
$$(3.4)$$

The solution to this equation which obeys the periodicity condition on the full circle and is antisymmetric across the fixed points of the $S^1/Z_2$ has a finite discontinuity at each of these points. Its derivative develops $\delta$-function singularities at $x^5 = 0, \pi\rho_0$, which cancel other $\delta$-function terms coming from the expansion of the formal 'square'. The regular part of the derivative, which is continuous everywhere, is

$$\partial_5 C_0 = \frac{1}{2\pi\rho_0}\sum_{m=1,2}\left(-\frac{\sqrt{2}\kappa^2}{\lambda^2}P^{(m)}\right.$$
$$\left. + \frac{1}{64\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}e^{\frac{9}{2}\sigma_m}(g_{55\,m})^{3/4}\bar{\epsilon}_m^{(4)}\epsilon_m^{(4)}\right). $$
$$(3.5)$$

where the subscript $m = 1, 2$ denotes the restriction of the given function to the $m$'th wall. We note that the coefficients of the gaugino bilinears above differ in higher order in $\xi_0$ (see (3.3)). The effective four-dimensional Lagrangian is obtained by substituting (3.5) into Lagrangian (3.2) and integrating over $x^5$.

We need to comment on the next-order corrections to the above solution. As we stressed earlier, higher-order corrections to (3.5) cannot be reliably calculated from the Lagrangian (3.2). It turns out that, in order to find the corrections reliably and to reconstruct the complete

four-dimensional Lagrangian at higher order in $\xi$, it would be necessary to go beyond the linear order in the $x^5$ dependence of the field $C_0$, hence beyond the order to which the effective Lagrangian can be trusted. Secondly, the compactification $11 \to 5$ leads in general to a non-linear $\sigma$-model structure which goes beyond the simple expression (3.2): as pointed out in [10], in five dimensions the 'perfect square' is a part of the larger non-linear $\sigma$-model structure. We reiterate that nonlinearities are to be expected in the solution of the full theory in five dimensions, since it contains nonlinear terms associated with gauging, as well as with the nonlinear $\sigma$ model structure.

It is useful to summarize certain properties of the zeroth-order result (3.5). First, we recall [9, 10] that the the order parameter for supersymmetry breaking in the microscopic five-dimensional vacuum is the nonvanishing vev of $\partial_5 C_0$. Secondly, (3.5) expresses this quantity in terms of fields charged under the gauge group, which are legitimate zero modes in the effective four-dimensional theory. We see that the terms corresponding to superpotentials generated on separate walls enter this expression additively, which happens to be the simplest possible structure leading to the effective 4d supergravity. Thirdly, the terms corresponding to gaugino bilinears enter the expression (3.5) with coefficients that are equal to the order to which we have solved the corresponding equation of motion.

In order to see how the soft terms found above would look in the presence of the non-universal hypermultiplets in the bulk, i.e., for Calabi-Yau spaces with Hodge number $h_{(2,1)} > 0$, we recall [10] that one has to make the replacement

$$2\partial_5 C_0 \to \frac{2}{1 - |Z^a|^2}\left(\partial_5 C_0 + \partial_5 C_a Z^a\right) \quad (3.6)$$

in the expressions for the soft terms, see [27], where $a = 1...h_{(2,1)}$ labels $Z_2$-odd $(C_a)$ and $Z_2$-even $(Z^a)$ scalars from the non-universal hypermultiplets. The presence of the derivatives $\partial_5 C_a$ signals that, in addition to the $F$ terms of the universal moduli $S, T$ there will be $F$ terms of the non-universal moduli participating in the breaking of the low-energy supersymmetry.

Further details leading to the microscopic, i.e. five-dimensional, derivation of the operators breaking softly the $N = 1, d = 4$ supersymmetry are presented in [27].

## 4. Conclusions

In this note we have presented observable supersymmetry breaking operators in perturbative nonstandard embeddings in Horava-Witten model of the effective low energy field theory limit of the strongly coupled heterotic string. The important feature which we studied in detail is that in these models charged matter lives in two sectors with different gauge kinetic functions i.e. on both 10d walls, which are in addition spatially separated. Also gaugino condensates may be present on both walls. We have studied how such two sectors combine with each other to form the effective supergravity in four dimensions. We have demonstrated how the soft supersymmetry breaking terms are born in the higher dimensional (5d) picture, and shown that the higher dimensional picture is consistent, at the level of lowest order solution to the equation of motion along the fifth dimension, with the effective four dimensional results. It has been demonstrated that at the lowest order in $\kappa^{2/3}$ the structure of the 4d supergravity with the gauge kinetic functions $f_{1,2} = S$ results from the integrating-out procedure. We have found nonuniversality between soft terms on different walls, which is due to the sign difference in the gauge kinetic functions and in the corrections to the kinetic functions of charged scalars.

We argue that the mixed $F^S/F^T$, and perhaps even the $F^S$-dominated, supersymmetry breaking should be possible in the class of models discussed here or in their counterparts in the more general constructions with different gauge sectors separated in higher dimensions.

Finally, we believe that present considerations should be helpful in more general situations, like type I/type IIB orientifolds models with gauge sectors located on different branes.

Detailed discussion of the issues raised in this note can be found in the papers [27] and [26].

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