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Theory Introduction to Lepton Flavour Violation

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In this proceedings, I give a brief theoretical overview on Lepton Flavour Violation (LFV). I list several lepton flavour changing transitions, discuss the current and future experimental searches, and mention some of the Beyond Standard Model scenarios that these experiments can probe. Finally, I discuss the phenomenology of LFV observables using Effective Field Theories.

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1. Introduction

Neutrino masses and oscillations provide conclusive evidence that the Standard Model, when defined with only left-handed neutrinos, is incomplete. With three generations $\alpha = e, \mu, \tau$ of leptons, divided into a SU(2) doublet $\ell_\alpha = (\nu_\alpha \; \alpha_L)$ and a SU(2) singlet $e_{R\alpha}$, with hypercharge $Y_\ell = -1/2$ and $Y_e = -1$ respectively, lepton flavour is defined by the Yukawa interactions with the Higgs doublet H

$$\mathcal{L}_{\rm Yuk} \supset \bar{\ell} Y_e H e + \text{h.c.} \tag{1}$$

These couplings are also responsible for the charged lepton masses when the electroweak symmetry is spontaneously broken, and are therefore diagonal in the lepton mass eigenstate basis. This leads to three accidental classical symmetries $U(1)_{L_\alpha}$ that correspond to the conservation of lepton flavour. Neutrino masses provide an alternative basis-choosing interaction, that regardless of the specifc model, must break all three symmetries consistently with the observed neutrino oscillations.

Since lepton flavor is not a fundamental symmetry of nature, transitions among charged leptons that violate flavor conservation—what we define as Lepton Flavor Violation (LFV)—are expected to occur. However, these transitions have (so far) never been observed and their rates remain unknown. If we minimally extend the Standard Model (SM) to account for neutrino masses by introducing a right-handed neutral partner for each neutrino flavor (thus giving them a Dirac mass), the predicted LFV rates are GIM-suppressed by the small neutrino masses ($Br_{\rm LFV} \sim G_F^2 m_\nu^4 \sim 10^{-50}$), rendering them practically unobservable. Observing a LFV transition would thus be a smoking-gun signal of New Physics (NP), potentially shedding light on the underlying mechanism behind neutrino masses. Moreover, accidental symmetries in the SM are easily violated when new states and interactions are introduced. Therefore, LFV processes provide a crucial probe of leptonic NP addressing both theoretical and experimental limitations of the SM. Several reviews on this topic are available in the literature [1–4].

In this short overview, we describe the main channels that are experimentally searched for and are predicted by a plethora of different Beyond SM (BSM) theories.

2. $\mu \to e$ transitions

Processes that violate $\mu \to e$ flavor are among the most sensitive probes of new physics. Low-energy searches for $\mu \to e$ transitions (such as $\mu \to e\gamma$ or $\mu \to e\bar{e}e$) benefit from the fact that muons are easy to produce and have sufficiently long lifetimes, allowing for extremely intense muon beams. These rare transitions, which have minimal background, can thus be probed with a sensitivity that scales with the experiments statistics.

The simplest possible LFV decay of the muon is $\mu \to e\gamma$, which is expected to occur in many BSM scenarios. Currently, non-observation of this process constrains the branching ratio to be $Br(\mu \to e\gamma) < 3.1 \times 10^{-13}$ (90% CL) [5], while future searches aim to reach branching ratio sensitivity of around $Br(\mu \to e\gamma) \sim 6 \times 10^{-14}$ [6]. The LFV radiative muon decay is predicted by various well-motivated extensions of the SM, including supersymmetry and grand-unified theories, extended scalar or gauge sectors, neutrino mass models, and many others (see [1, 3, 4] and references therein for more details). Regardless of the specific model, the decay amplitude of this process can

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be cast in the following general form

$$iM(\mu \to e\gamma) = 2m_\mu q_\alpha \epsilon^*_\beta \bar{u}_e(p_e)(A_L P_L + A_R P_R)\sigma^{\alpha\beta} u_\mu(p_e + q) \tag{2}$$

where $P_{L,R}$ are the left-handed and right-handed projector, $\sigma_{\alpha\beta} = i/2\left[\gamma_\alpha, \gamma_\beta\right]$, q is the 4-momentum of the outgoing photon, and $A_{D,X}$ are dimensionful functions of the model couplings and masses. The dipole transitions is chirality-flipping, hence we conveniently factorize the muon mass to stress that a mass insertion in the fermion line is needed. The following amplitude yields a branching ratio

$$Br(\mu \to e\gamma) = \frac{48\pi^2}{G_F^2}(|A_L|^2 + |A_R|^2 + \dots) \tag{3}$$

where G_F is the Fermi constant, and we have neglected the interference terms between amplitudes with different electron chiralities, because they are suppressed by the small electron mass.

Another possible LFV transitions involving muons is the three body decay $\mu^\pm \to e^\pm e^+ e^-$ ($\mu \to 3e$ in short). The current bound on the branching ratio for this decay, set by the SINDRUM collaboration, is $Br(\mu^\pm \to e^\pm e^+ e^-) < 10^{-12}$. Anticipated improvements in future searches are expected to enhance this limit by four orders of magnitude [8]. If the $\mu \to e\gamma$ transition is allowed within a model, then $\mu \to 3e$ can also occur at the same loop level, because it is always possible to attach a flavor-conserving electron-positron current to the photon. Assuming that this contribution dominates the $\mu \to 3e$ decay, we find that

$$Br(\mu \to 3e) \sim \frac{\alpha_{\rm em}}{3\pi}\left(2\log\left(\frac{m_\mu}{m_e}\right) - \frac{11}{4}\right) Br(\mu \to e\gamma), \tag{4}$$

This expression implies that the current (future) upper limit on $\mu \to e\gamma$ would yield $Br(\mu \to 3e) \lesssim 10^{-15}$. We should stress that there may be other contribution to $\mu \to 3e$ that are not related to the dipole, as for instance is the case in the type-II seesaw, where the scalar triplet can mediate the process at the tree level [9]. Moreover, upcoming searches for $\mu \to 3e$ target branching ratio sensitivities on the order of $\sim 10^{-16}$, and they have the potential to explore values even smaller than those predicted by Eq. (4), suggesting sensitivity to dipole coefficients that are too small to be observable in $\mu \to e\gamma$.

Lastly in this section, we discuss the process known as $\mu \to e$ conversion in nuclei, which constitute another promising channel that expects a significant improvement in the upcoming experimental searches. A muon, when stopped in a material, can form a muonic atom with a nucleus of the target. While in a bound state, the muon can undergo two SM processes: decay in orbit, where an electron and an (anti-)neutrino are emitted, or muon capture, given by

$$\mu^- N(A, Z) \to \nu_\mu N'(A, Z - 1) \tag{5}$$

where A, Z are, respectively, the mass and atomic number of the nucleus N. In the presence of LFV interactions that change muons to electrons, a muon can be captured by the nucleus without the emission of a neutrino

$$\mu^- N(A, Z) \to e^- N(A, Z). \tag{6}$$

in the process known as $\mu \to e$ conversion in nuclei. After cascading down in energy levels, the ground state of the muonic atom is a 1s orbital with a binding energy E_b, and in the final

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state a monochromatic electron with energy $\sim m_\mu - E_b$ is emitted while the nucleus recoils. The SINDRUMII collaboration sets the upper limit $\Gamma(\mu Au \to eAu)/\Gamma_{capt} < 7 \times 10^{-13}$ [10] on the rate of $\mu \to e$ conversion with respect to the flavour conserving muon capture (on a gold target). Upcoming searches led by the Mu2e and COMET collaboration aim at sensitivities of order $Br(\mu Al \to eAl) \sim 10^{-17}$, using Aluminum as a target [11, 12].

The calculation of the $\mu \to e$ conversion rate can be found in [1]. The spin-independent rate of this process is a coherent sum of the LFV interactions with each nucleon inside the nuclei. Therefore this rate is enhanced by the squared atomic number Z^2, making the process an extremely sensitive probe of LFV interactions involving quarks. Such interactions are expected, for instance, in the presence of leptoquarks or extra gauge bosons arising from gauge symmetries under which leptons and quarks are charged. Additionally, like in $\mu \to 3e$, we expect contributions from the $\mu \to e$ dipole, and future searches may become the most sensitive probe of $\mu \to e\gamma$ interactions[1].

3. $\tau \to l$ ($l = e, \mu$) transitions

Lepton flavor violation can also manifest in the decay of the τ lepton. Taus are significantly heavier and more short-lived than muons, making tau beams physically unfeasible. Searches for τ flavor-changing decays are performed at colliders, especially at B factories where a large number of τ pairs are produced. Although the sensitivities of these searches do not match those of muon facilities due to reduced statistics, τ LFV decays offer orthogonal tests for various BSM scenarios, and models exist favoring τ LFV over $\mu \to e$. This is for instance expected in the presence of new states coupling predominantly with third generation fermions[2], or in flavour models that respect an approximate lepton triality symmetry which forbids $\mu \to e$ process but allows for some $\tau \to l$ transitions [14].

Future searches may further constrain or potentially discover lepton flavor changing new physics. The Belle-II experiment aims to enhance the sensitivity of branching ratios for several transitions by up to two orders of magnitude, with $Br(\tau \to l + \dots) \lesssim 10^{-8} \to 10^{-10}$. Figure 1 illustrates the projected branching fractions for a multitude of different flavour changing τ decay channels, alongside the current upper limits established by various experiments. The large available phase space make τ LFV decays an exciting avenue for discovering new physics. This is because, with the numerous processes that can be investigated, models can be over-constrained and thus distinguished. Moreover, the potentially observables branching ratios are quite large, implying that τ LFV physics should be relatively simple if ever observed, considering that experiments are (with some exceptions) insensitive to loop contributions.

4. LFV decays of heavy particles

Another class of possible LFV channels is the decay of non-leptonic particles into a final state with non-zero lepton flavour. These include the decay of heavy bosons, such as the Z and h, or the decay of mesons and baryons.

[1] While we can discuss the sensitivity, which represents the smallest experimentally detectable value, the potential presence of additional contributions prevents us from using $\mu \to e$ conversion searches to place model-independent bounds on $\mu \to e\gamma$ interactions.

[2] With the addition of small LFV rotations between the third and lighter generations

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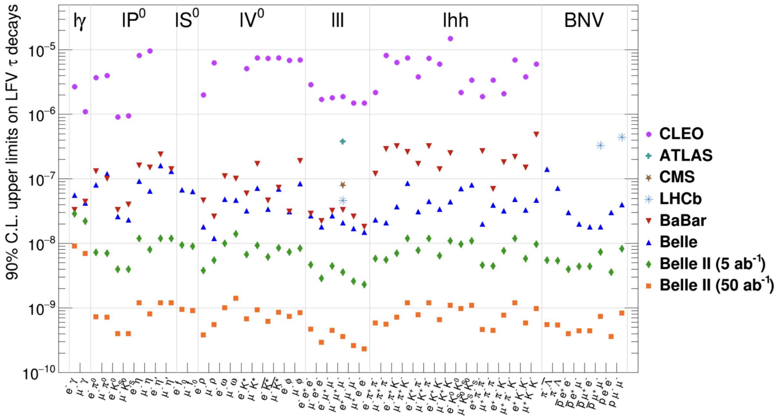

Figure 1: Figure taken from [13] showing the current and upcoming branching ratio sensitivities on τ LFV decays.

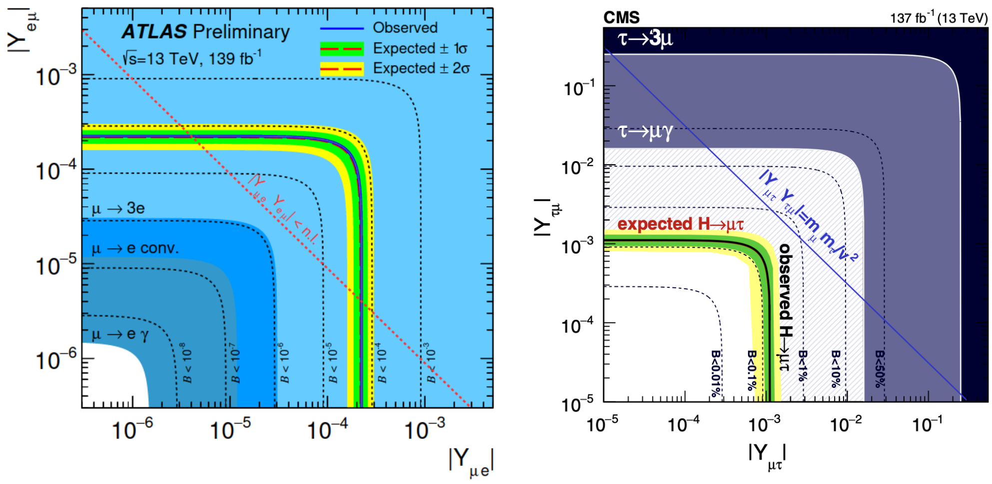

Figure 2: Left figure from [16] and right figure from [17], showing the observed limits on $(Y_e)_{ij,ji}$ arising from $h \longrightarrow l_i^{\pm} l_j^{\mp}$ compared with the sensitivities of the low-energy probes.

LFV decays of the Higgs boson are, for instance, possible in models with extra doublets when no additional symmetries forbid flavour changing neutral currents. One can introduce general off-diagonal Yukawa couplings for the SM Higgs

$$\mathcal{L}^{h}_{\rm LFV} = -\frac{(Y_e)_{ij}}{\sqrt{2}} \bar{l}_i h P_R l_i + \text{h.c} \tag{7}$$

which can be directly probed by $h \longrightarrow l_i^{\pm} l_j^{\mp}$ searches, but can also contribute to low-energy processes via loops [15]. Figure 2 shows the upper limits on the $\mu \leftrightarrow e$ and $\tau \leftrightarrow \mu$ ($\tau \leftrightarrow e$ is similar) off-diagonal Yukawas arising from the LFV decay Higgs decay searches at the LHC, and these limits are compared with the sensitivities of other low-energy processes. We can observe that $\mu \longrightarrow e$ transitions are generally sensitive to smaller Yukawa couplings than $h \longrightarrow e^{\pm}\mu^{\mp}$, but these are not strictly constrained because cancellations with different contributions are possible. In the

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$\tau \leftrightarrow l$ sector we instead find that $h \to \tau^\pm l^\mp$ are the most sensitive probes of the LFV Yukawas. Similar conclusions hold for the decay $Z \to l_i^\pm l_j^\mp$.

Hadron decays can be a probe of LFV interaction between leptons and quarks. These can be neutral current transitions, such as $K \to e^\pm\mu^\mp$ or $B \to K\tau^\pm\mu^\mp$, but also charged current processes like $B \to \tau\nu_e$. The latter type of transitions may also have a SM background, because the neutrino flavour is not identified, and upper limits on these interactions can be imposed by the non-observed deviations from the SM value. LFV decays of hadrons are expected in supersymmetry with R-parity violating couplings [18], or in models with leptoquarks[3] [19].

5. Effective Field Theory for LFV

Under the assumption that the LFV physics is heavy, i.e $\Lambda_{\rm NP} \gtrsim$ TeV, Effective Field Theories (EFTs) provide a powerful model-independent framework in which we can describe LFV interactions. In the EFT, the UV physics is integrated out and can be parametrised by the coefficients of contact interactions among the light degrees of freedom

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{d\leq 4} + \sum_{I,n>4} \frac{C_n^I O_n^I}{\Lambda_{\rm NP}^{n-4}} \tag{8}$$

which are further suppressed by powers of the heavy scale according to the operator dimension. Observables can be computed with the non-renormalizable operators, and experimental null-results can be translated in upper limits on the size of their coefficients. For instance, the $\mu \to e\gamma$ operator

$$\delta\mathcal{L}_{\mu\to e\gamma} = \frac{m_\mu}{\Lambda_{\rm NP}^2}(C_{D,L}\bar{e}\sigma_{\alpha\beta}P_L\mu + C_{D,R}\bar{e}\sigma_{\alpha\beta}P_R\mu)F^{\alpha\beta} \tag{9}$$

would yield the amplitude in Eq. (2) with $A_X = (C_{D,X}/\Lambda_{\rm NP}^2)$, so that the branching ratio for $\mu \to e\gamma$ reads

$$Br(\mu \to e\gamma) = 48\pi^2 \left(\frac{1}{G_F^2\Lambda_{\rm NP}^4}\right)(|C_{D,L}|^2 + |C_{D,R}|^2) < 3.1\times 10^{-13} \to \frac{|C_{D,Y}|}{G_F\Lambda_{\rm NP}^2} \lesssim 10^{-8} \tag{10}$$

Operator coefficients, like any coupling in a quantum field theory, run with the renormalization scale when the operators are dressed with SM loops, as described by their Renormalization Group Equations (RGEs). Observables are sensitive to the operator coefficients at the energy of the experiments, hence, they are also sensitive to all high-energy operators that can efficiently mix with those in the RGEs. Therefore, LFV observables are capable of probing a wide variety of flavour changing operators thanks to the operator mixing. There are numerous RGE-improved effective analysis of LFV transitions, including $\mu \to e$ processes that are otherwise flavour diagonal [20, 21], τ decays [22], and semileptonic transitions [23].

For EFT calculations to be entirely model-independent, they should be performed from the bottom up. This means that observables are calculated in the most general EFT that is consistent with the symmetries, incorporating every contribution that could be within the reach of the experiments

[3] These are particularly motivated in the B sector, where some experimental anomalies may be hinting at new physics in rare B meson decays

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[24]. A bottom-up calculation translates the experimental data at low energy into a combination of operator coefficients at the heavy physics scale, and it identifies the region of coefficient space where models should sit [25].

From this perspective, in the event of a LFV discovery in future experiments, it would be theoretically possible to exclude models by identifying how they populate the experimentally accessible regions. If we find parts of parameter space that models cannot reach, an observation in those regions would rule them out. This approach has been followed in [26, 27]. These works demonstrate that upcoming $\mu \to e$ searches have the potential to distinguish among some popular BSM models.

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